# Problemas de límites 2º de Bachillerato

**Abril 2002**

**Problema 1** Hallar el dominio y recorrido de las siguientes funciones:

1. $f(x) = \sqrt{x-1}$
2. $f(x) = x^2$
3. $f(x) = \sqrt{9-x^2}$
4. $f(x) = \frac{1}{|x|}$
5. $f(x) = \frac{|x|}{x}$
6. $f(x) = \sqrt{1-x}$
7. $f(x) = 4 - x^2$
8. $f(x) = \sqrt{25-x^2}$
9. $f(x) = |x-2|$
10. $f(x) = \sqrt{x^2-4}$

**Problema 2** Calcular los siguientes límites:

1. $\lim_{x\to -1} \dfrac{x^2-1}{x+1}$
2. $\lim_{x\to -1} \dfrac{2x^2-x-3}{x+1}$
3. $\lim_{x\to 3} \dfrac{x-3}{x^2-9}$
4. $\lim_{x\to -1} \dfrac{x^3+1}{x+1}$
5. $\lim_{x\to -2} \dfrac{x^3+8}{x+2}$
6. $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-1}$
7. $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$
8. $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

9. $\lim_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

10. $\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3}$

11. $\lim_{x\to 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}$

12. $\lim_{x\to 1} \frac{1-x}{\sqrt{5-x^2} - 2}$

13. $\lim_{x\to 2} \frac{x^5 - 32}{x-2}$

14. $\lim_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

15. $\lim_{x\to 0} \frac{\operatorname{sen}\ x}{5x}$

16. $\lim_{x\to 0} \frac{(1-\cos x)}{x}$

17. $\lim_{x\to 0} \frac{\sec x - 1}{x \sec x}$

18. $\lim_{x\to 0} \frac{\cos x \tan x}{x}$

19. $\lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x}$

20. $\lim_{x\to \pi} x \sec x$

21. $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x}$

22. $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\operatorname{sen} x - \cos x}$

23. $\lim_{t\to 0} \frac{\operatorname{sen}^2 t}{t^2}$. (Ayuda: $(\frac{\operatorname{sen} t}{t})^2 = \frac{\operatorname{sen}^2 t}{t^2}$)

24. $\lim_{t\to 0} \frac{\operatorname{sen} 3t}{t}$. (Ayuda: $\frac{\operatorname{sen} 3t}{t} = 3(\frac{\operatorname{sen} 3t}{3t})$)

25. $\lim_{t\to 0} \frac{\operatorname{sen} 2t}{\operatorname{sen} 3t}$. (Ayuda: $\frac{\operatorname{sen} 2t}{\operatorname{sen} 3t} = \frac{2}{3} \cdot \frac{\operatorname{sen} 2t}{2t} \cdot \frac{3t}{\operatorname{sen} 3t}$)

26. $\lim_{x\to 0} \frac{\tan^2 x}{x}$

27. $\lim_{h\to 0} \dfrac{(1-\cos h)^2}{h}$

**Problema 3** Calcular los límites siguientes:

1. $\lim_{x\to\infty} \dfrac{2x-1}{3x+2}$
2. $\lim_{x\to\infty} \dfrac{5x^3+1}{10x^3-3x^2+7}$
3. $\lim_{x\to\infty} \dfrac{x}{x^2-1}$
4. $\lim_{x\to\infty} \dfrac{2x^{10}-11}{10x^1 1-3}$
5. $\lim_{x\to\infty} \dfrac{5x^2}{x+3}$
6. $\lim_{x\to\infty} \dfrac{x^4-2x^2+3x+1}{x^2-3x+2}$
7. $\lim_{x\to\infty} (2x-\dfrac{1}{x^2})$
8. $\lim_{x\to\infty} (x+3)^{-2}$
9. $\lim_{x\to\infty} (\dfrac{2x}{x-1}+\dfrac{3x}{x+1})$
10. $\lim_{x\to\infty} (\dfrac{2x^2}{x-1}+\dfrac{3x}{x+1})$
11. $\lim_{x\to\infty} (x+\sqrt{x^2+3})$
12. $\lim_{x\to\infty} (2x-\sqrt{4x^2+1})$
13. $\lim_{x\to\infty} (x-\sqrt{x^2+x})$
14. $\lim_{x\to\infty} (3x+\sqrt{9x^2-x})$
15. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2-x}}$
16. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$
17. $\lim_{x\to\infty} \dfrac{2x+1}{\sqrt{x^2-x}}$

18. $\lim_{x\to\infty} \dfrac{-3x+1}{\sqrt{x^2+x}}$

19. $\lim_{x\to\infty} \dfrac{x^2-x}{\sqrt{x^4+1x}}$

20. $\lim_{x\to\infty} \dfrac{2x}{\sqrt{4x^2+1}}$

21. $\lim_{x\to\infty} \dfrac{\operatorname{sen} 2x}{x}$

22. $\lim_{x\to\infty} \dfrac{1}{2x+\operatorname{sen} x}$

23. $\lim_{x\to\infty} \operatorname{sen}\dfrac{1}{x}$

24. $\lim_{x\to\infty} x \tan\dfrac{1}{x}$

**Problema 4** Calcular por la regla de L'Hôpital los límites de las siguientes funciones:

1. $\lim_{x\to 2} \dfrac{x^2-x-2}{x-2}$

2. $\lim_{x\to -1} \dfrac{x^2-x-2}{x+1}$

3. $\lim_{x\to 0} \dfrac{\sqrt{4-x^2}-2}{x}$

4. $\lim_{x\to 2^-} \dfrac{\sqrt{4-x^2}}{x-2}$

5. $\lim_{x\to 0} \dfrac{e^x-(1-x)}{x}$

6. $\lim_{x\to 0^+} \dfrac{e^x-(1+x)}{x}$

7. $\lim_{x\to 1} \dfrac{\ln x}{x^2-1}$

8. $\lim_{x\to\infty} \dfrac{\ln x}{x}$

9. $\lim_{x\to\infty} \dfrac{3x^2-2x+1}{2x^2+3}$

10. $\lim_{x\to\infty} \dfrac{e^x}{x}$

11. $\lim_{x\to\infty} \dfrac{x-1}{x^2+2x+3}$

12. $\lim_{x\to\infty} \dfrac{x^2+2x+3}{x-1}$

13. $\lim_{x\to\infty} \dfrac{x^2}{e^x}$

14. $\lim_{x\to 0^+} x^2 \ln x$

15. $\lim_{x\to 0}(\dfrac{1}{x} - \dfrac{1}{x^2})$

16. $\lim_{x\to 2}(\dfrac{8}{x^2-4} - \dfrac{x}{x-2})$

17. $\lim_{x\to 2}(\dfrac{1}{x^2-4} - \dfrac{\sqrt{x-1}}{x^2-4})$

18. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$

19. $\lim_{x\to 1^+}(\dfrac{3}{\ln x} - \dfrac{2}{x-1})$

20. $\lim_{x\to 0^+} x^{1/x}$

21. $\lim_{x\to 0^+} (e^x + x)^{1/x}$

22. $\lim_{x\to\infty} x^{1/x}$

23. $\lim_{x\to\infty} (1+\dfrac{1}{x})^x$

24. $\lim_{x\to\infty} (1+x)^{1/x}$

25. $\lim_{x\to\pi} \dfrac{\operatorname{sen} x}{x-\pi}$

26. $\lim_{x\to 0} \dfrac{\operatorname{sen} 2x}{\operatorname{sen} 3x}$

27. $\lim_{x\to 0} \dfrac{\operatorname{sen} ax}{\operatorname{sen} bx}$

28. $\lim_{x\to 0} x \operatorname{cosec} x$

29. $\lim_{x\to 0} x^2 \cot x$

30. $\lim_{x\to\infty} (x \operatorname{sen}\dfrac{1}{x})$

31. $\lim_{x\to\infty}(x\ \tan\frac{1}{x})$

32. $\lim_{x\to 0}\frac{\operatorname{arcsen} x}{x}$

33. $\lim_{x\to 1}\frac{\arctan x-\frac{\pi}{4}}{x-1}$