Problemas de integrales
2º de Bachillerato

Comprueba el valor de las siguientes integrales:

1. $\displaystyle\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

2. $\displaystyle\int \frac{3}{x^2+x-2}\,dx = \ln\left|\frac{x-1}{x+2}\right| + C$

3. $\displaystyle\int \frac{1}{4x^2-9}\,dx = \frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right| + C$

4. $\displaystyle\int \frac{x+1}{x^2+4x+3}\,dx = \ln|x+3| + C$

5. $\displaystyle\int \frac{5-x}{2x^2+x-1}\,dx = \frac{3\ln|2x-1| - 4\ln|x+1|}{2} + C$

6. $\displaystyle\int \frac{3x^2-7x-2}{x^3-x}\,dx = \frac{1}{2}\left(\ln|x^4(x^2-1)| - 7\ln\left|\frac{x-1}{x+1}\right|\right) + C$

7. $\displaystyle\int \frac{x^2+12x+12}{x^3-4x}\,dx = \ln\left|\frac{(x-2)^5}{x^3(x+2)}\right| + C$

8. $\displaystyle\int \frac{x^3-x+3}{x^2+x-2}\,dx = \ln|(x-1)(x+2)| + \frac{x^2}{2} - x + C$

9. $\displaystyle\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx = \frac{1}{2}\left(\ln\left|\frac{(x-4)^3}{x+2}\right| + 2x^2\right) + C$

10. $\displaystyle\int \frac{x+2}{x^2-4x}\,dx = \frac{1}{2}\ln\left|\frac{(x-4)^3}{x}\right| + C$

11. $\displaystyle\int \frac{4x^2+2x-1}{x^3+x^2}\,dx = \ln|x^3(x+1)| + \frac{1}{x} + C$

12. $\displaystyle\int \frac{2x-3}{(x-1)^2}\,dx = \frac{1}{x-1} + 2\ln|x-1| + C$

13. $\displaystyle\int \frac{x^4}{(x-1)^3}\,dx = 6\ln|x-1| - \frac{8x-7}{2(x-1)^2} + \frac{x^2}{2} + 3x + C$

14. $\displaystyle\int \frac{4x^2-1}{2x(x^2+2x+1)}\,dx = \frac{1}{2}\ln\left|\frac{(x+1)^5}{x}\right| + \frac{3}{2(x+1)} + C$

15. $\displaystyle\int \frac{3x}{x^2-6x+9}\,dx = 3\ln|x-3| - \frac{3x}{x-3} + C$

16. $\int \dfrac{6x^2+1}{x^2(x-1)^3}\,dx = 3\ln\left|\dfrac{x-1}{x}\right| + \dfrac{4x-11}{2(x-1)^2} + \dfrac{1}{x} + C$

17. $\int \dfrac{x^2-1}{x^3+x}\,dx = -\ln\left|\dfrac{x}{x^2+1}\right| + C$

18. $\int \dfrac{x}{x^3-1}\,dx = \dfrac{\sqrt{3}}{3}\arctan\left(\dfrac{\sqrt{3}}{3}(2x+1)\right) - \dfrac{1}{6}\ln\left|\dfrac{x^2+x+1}{(x-1)^2}\right| + C$

19. $\int \dfrac{x^2}{x^4-2x^2-8}\,dx = \dfrac{\sqrt{2}}{6}\arctan\left(\dfrac{\sqrt{2}}{2}\cdot x\right) + \dfrac{1}{6}\ln\left|\dfrac{x-2}{x+2}\right| + C$

20. $\int \dfrac{2x^2+x+8}{(x^2+4)^2}\,dx = \arctan\left(\dfrac{x}{2}\right) - \dfrac{1}{2(x^2+4)} + C$

21. $\int \dfrac{x}{16x^4-1}\,dx = \dfrac{1}{16}\ln\left|\dfrac{4x^2-1}{4x^2+1}\right| + C$

22. $\int \dfrac{x^2-4x+7}{x^3-x^2+x+3}\,dx = \dfrac{1}{2}\ln\left|\dfrac{(x+1)^4}{x^2-2x+3}\right| + C$

23. $\int \dfrac{x^2+x+2}{(x^2+2)^2}\,dx = \dfrac{\sqrt{2}}{2}\arctan\left(\dfrac{\sqrt{2}}{2}\cdot x\right) - \dfrac{1}{2(x^2+2)} + C$

24. $\int \dfrac{x^3}{(x^2-4)^2}\,dx = \dfrac{1}{2}\left(\ln|x^2-4|\right) - \dfrac{x^2}{x^2-4} + C$

25. $\int \dfrac{x^2+5}{x^3-x^2+x+3}\,dx = \sqrt{2}\arctan\left(\dfrac{\sqrt{2}}{2}\cdot(x-1)\right) + \ln|x+1| + C$

26. $\int \dfrac{x^2+x+3}{x^4+6x^2+9}\,dx = \dfrac{\sqrt{3}}{3}\arctan\left(\dfrac{\sqrt{3}}{3}\cdot x\right) - \dfrac{1}{2(x^2+3)} + C$

27. $\int \dfrac{6x^2-3x+14}{x^3-2x^2+4x-8}\,dx = \dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + \ln|x^2+4| + 4\ln|x-2| + C$

28. $\int \dfrac{x(2x-9)}{x^3-6x^2+12x-8}\,dx = 2\ln|x-2| + \dfrac{x+3}{(x-2)^2} + C$

29. $\int \dfrac{2x^2-2x+3}{x^3-x^2-x-2}\,dx = -\sqrt{3}\arctan\left(\dfrac{\sqrt{3}}{3}\cdot(2x+1)\right) + \dfrac{\ln|x^2+x+1|}{2} +$

$\qquad + \ln|x-2| + C$

30. $\displaystyle\int \frac{-x^4 + 5x^3 - 9x^2 + 3x - 6}{x^5 - x^4 - x + 1}\, dx = \arctan x + 2\ln|x-1| - 3\ln|x+1| +$

$\displaystyle + \frac{2}{x-1} + C$

31. $\displaystyle\int \frac{3}{2x^2 + 5x + 2}\, dx = \ln\left|\frac{2x+1}{x+2}\right| + C$

32. $\displaystyle\int \frac{1}{x^2 - 4}\, dx = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$

33. $\displaystyle\int \frac{x+1}{x(x^2+1)}\, dx = \arctan x + \frac{1}{2}\ln\left|\frac{x^2}{x^2+1}\right| + C$

34. $\displaystyle\int \frac{x-1}{x^2(x+1)}\, dx = \frac{1}{x} - 2\ln\left|\frac{x+1}{x}\right| + C$

35. $\displaystyle\int \frac{x^4 + 4x^3 - x^2 + 5x + 1}{x^5 + x^4 + x^3 - x^2 - 2}\, dx = 3\arctan(x+1) - \arctan x + \ln|x-1| + C$

36. $\displaystyle\int \frac{x^2 - x}{x^2 + x + 1}\, dx = x - \ln|x^2 + x + 1| + C$

37. $\displaystyle\int \frac{\sin x}{\cos x(\cos x - 1)}\, dx = -\ln\left|\frac{\cos x - 1}{\cos x}\right| + C$

hacer el cambio $u = \cos x$.

38. $\displaystyle\int \frac{\sin x}{\cos x + \cos^2 x}\, dx = \ln\left|\frac{1}{\cos x} + 1\right| + C$

hacer el cambio $u = \cos x$.

39. $\displaystyle\int \frac{e^x}{(e^x - 1)(e^x + 4)}\, dx = \frac{1}{5}\ln\left|\frac{e^x+1}{e^x+4}\right| + C$

hacer el cambio $u = e^x$.

40. $\displaystyle\int \frac{e^x}{(e^{2x} + 1)(e^x - 1)}\, dx = -\frac{1}{2}\arctan e^x + \frac{1}{4}\ln\left|\frac{(e^x-1)^2}{e^{2x}+1}\right| + C$

hacer el cambio $u = e^x$.

41. $\displaystyle\int \frac{3\cos x}{\sin^2 x + \sin x - 2}\, dx = \ln\left|\frac{1 - \sin x}{\sin x + 2}\right| + C$

hacer el cambio $u = \sin x$.