Problemas de Matemáticas 4º de ESO
Logaritmos

Resuelve las siguientes expresiones logarítmicas:

1. $\log(2x) = -1$
   Solución:
   $$2x = 10^{-1} \Longrightarrow \frac{1}{20}$$

2. $\log x^2 = \dfrac{1}{2}$
   Solución:
   $$x^2 = 10^{\frac{1}{2}} \Longrightarrow x = \sqrt{10^{\frac{1}{2}}} = (10^{\frac{1}{2}})^{\frac{1}{2}} = 10^{\frac{1}{4}}$$
   $$x = \sqrt[4]{10}$$

3. $\log(3x) = 2$
   Solución:
   $$3x = 10^2 \Longrightarrow x = \frac{100}{3}$$

4. $\log x = -1$
   Solución:
   $$x = 10^{-1} = \frac{1}{10}$$

5. $\log x = \dfrac{3}{2}$
   Solución:
   $$x = 10^{\frac{3}{2}} = \sqrt{10^3} = 10\sqrt{10}$$

6. $\log(3x^2) = -2$
   Solución:
   $$3x^2 = 10^{-2} \Longrightarrow x^2 = \frac{1}{300}$$
   $$x = \sqrt{\frac{1}{300}} = \frac{1}{\sqrt{300}} = \frac{1}{10\sqrt{3}} = \frac{\sqrt{3}}{10(\sqrt{3})^2} = \frac{\sqrt{3}}{30}$$

7. $\log x + \log 50 = \log 100$
   Solución:
   $$\log(x \cdot 50) = \log 1000$$
   $$50x = 1000 \Longrightarrow x = 20$$

8. $\log x = 1 + \log(22 - x)$
   Solución:
   $$\log x = \log 10 + \log(22 - x) \Longrightarrow \log x = \log 10(22 - x)$$
   $$x = 10(22 - x) \Longrightarrow x = 220 - 10x \Longrightarrow 11x = 220 \Longrightarrow x = \frac{220}{11} = 20$$

9. $2 \log x - \log(x - 16) = 2$
   Solución:

$$\log x^2 - \log(x - 16) = \log 100 \implies \log \frac{x^2}{x - 16} = \log 100$$

$$\frac{x^2}{x - 16} = 100 \implies x^2 = 100(x - 16) \implies x^2 = 100x - 1600 \implies$$

$$x^2 - 100x + 1600 = 0 \implies x = 80, \;\; x = 20$$

10. $\log x^3 = \log 6 + 2 \log x$
    Solución:

$$\log x^3 = \log 6 + \log x^2 \implies \log x^3 = \log(6x^2)$$

$$x^3 = 6x^2 \implies x^3 - 6x^2 = 0 \implies x^2(x - 6) = 0 \implies x = 0, \;\; x = 6$$

La solución $x = 0$ no es válida.

11. $3 \log x + 2 \log x^2 = \log 128$
    Solución:

$$\log x^3 + \log x^4 = \log 128 \implies \log x^7 = \log 128$$

$$x^7 = 128 \implies x^7 = 2^7 \implies x = 2$$

12. $5 \log(2x) = 20$
    Solución:

$$\log(2x) = 4 \implies 2x = 10^4 = 10000 \implies x = 5000$$

13. $\log \left(\frac{2x - 4}{5} \right) = 2$
    Solución:
$$\log \left(\frac{2x - 4}{5} \right) = \log 100$$

$$\frac{2x - 4}{5} = 100 \implies 2x - 4 = 500 \implies 2x = 504 \implies x = 252$$

14. $\log(7x + 15) - \log 5 = 1$
    Solución:
$$\log \frac{7x + 15}{5} = \log 10$$

$$\frac{7x + 15}{5} = 10 \implies 7x + 15 = 50 \implies 7x = 35 \implies x = 5$$

15. $\log \dfrac{x}{2} = 1 + \log(21 - x)$

    Solución:

    $$\log \frac{x}{2} = \log 10 + \log(21 - x) \Longrightarrow \log \frac{x}{2} = \log 10(21 - x)$$

    $$\frac{x}{2} = 10(21 - x) \Longrightarrow x = 20(21 - x) \Longrightarrow x = 420 - 20x \Longrightarrow$$

    $$21x = 420 \Longrightarrow x = 20$$

16. $\log \dfrac{10}{x} = 2 - 2\log x$

    Solución:

    $$\log \frac{10}{x} = \log 100 - \log x^2 \Longrightarrow \log \frac{10}{x} = \log \frac{100}{x^2}$$

    $$\frac{10}{x} = \frac{100}{x^2} \Longrightarrow 10x^2 = 100x \Longrightarrow 10x^2 - 100x = 0$$

    $$\Longrightarrow 10x(x - 10) = 0 \Longrightarrow x = 0, \ \ x = 10$$

    La solución $x = 0$ no es válida.

17. $2\log x - \log(x^2 - 2x + 6) = 0$

    Solución:

    $$\log x^2 = \log(x^2 - 2x + 6)$$

    $$x^2 = x^2 - 2x + 6 \Longrightarrow x = 3$$

18. $\log(2x - 3) + \log(3x - 2) = 2 - \log 25$

    Solución:

    $$\log(2x-3)\cdot(3x-2) = \log 100 - \log 25 \Longrightarrow \log(2x-3)\cdot(3x-2) = \log \frac{100}{25}$$

    $$(2x - 3) \cdot (3x - 2) = \frac{100}{25} \Longrightarrow 6x^2 - 13x + 2 = 0 \Longrightarrow x = 2, \ \ x = \frac{1}{6}$$

    La solución $x = \dfrac{1}{6}$ no es válida.

19. $\log x + \log 20 = 3$

    Solución:

    $$\log(20x) = \log 1000$$

    $$20x = 1000 \Longrightarrow x = 50$$

20. $2\log x = \log(10 - 3x)$

    Solución:

    $$\log x^2 = \log(10 - 3x)$$

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    $$x^2 = 10 - 3x \Longrightarrow x^2 + 3x - 10 = 0 \Longrightarrow x = -5, \ \ x = 2$$

    La solución $x = -5$ no es válida.

21. $3\log(5x) = -9$
    Solución:

$$\log(5x) = -\frac{9}{3} = -3 \implies 5x = 10^{-3} \implies x = \frac{1}{5000} = 0,0002$$

22. $\log(x+1)^2 = 2$
    Solución:

$$(x+1)^2 = 10^2 \implies x^2 + 2x - 99 = 0 \implies x = 9, \;\; x = -11$$