Problemas de Matemáticas 4º de ESO
Octubre 2002

1 Ecuaciones Logarítmicas:

1. $5 \log 2x = 20$ Sol: $x = 5000$

2. $3 \log 5x = -9$ Sol: $x = 0,0002$

3. $\log \frac{2x-4}{5} = 2$ Sol: $x = 252$

4. $\log(x+1)^2 = 2$ Sol: $x = 9;\ x = -11$

5. $\log(7x+15) - \log 5 = 1$ Sol: $x = 5$

6. $\log \frac{x}{2} = 1 + \log(21 - x)$ Sol: $x = 20$

7. $\log \frac{10}{x} = 2 - 2 \log x$ Sol: $x = 10;\ x = 0$

8. $2 \log x - \log(x^2 - 2x + 6) = 0$ Sol: $x = 3$

9. $\log(2x - 3) + \log(3x - 2) = 2 - \log 25$ Sol: $x = 2;\ x = \frac{1}{6}$

10. $\log(3x^2 - 2) = 1 + \log(x - 1)$ Sol: $x = 2;\ x = \frac{4}{3}$

11. $\log x^2 + 3 \log x = 2$ Sol: $x = 10^{\frac{2}{5}}$

12. $2 \log x^2 - 2 \log x = 2$ Sol: $x = 10$

13. $\log x^2 + 1 = \log x^3$ Sol: $x = 10$

14. $\log(1 - x) + \log x = 1$ Sol: No tiene solución real.

15. $\log x - \log(1 - x) = 1$ Sol: $x = \frac{10}{11}$

16. $\log x + 2 = \log x^3$ Sol: $x = 10$

17. $\log(1 + x) + \log(1 - x) = 2$ Sol: No tiene solución real.

18. $\log(2x + 7) - \log(x - 1) = \log 5$ Sol: $x = 4$

19. $\frac{\log(35 - x^2)}{\log(5 - x)} = 3$ Sol: $x = 3 :\ x = 2$

20. $\log x^2 - \log \frac{10x + 11}{10} = 1$ Sol: $x = 11;\ x = -1$

21. $\log(2x + 2) + \log(x + 3) = \log 6$ Sol: $x = 0,\ x = -4$

22. $\frac{\log 2 + \log(x^2 - 2)}{\log(2x - 2)} = 2$ Sol: $x = 2$

23. $\log(x+6) - \frac{1}{2}\log(2x-3) = 2 - \log 25$ Sol: $x = 6; \quad x = 14$

24. $\log x = \log 2 + 2\log(x-3)$ Sol: $x = \frac{9}{2}; \quad x = 2$

25. $2\log x = 2 + \log x$ Sol: $x = 0; \quad x = 2$

26. $\log 8 + (x^2 - 5x + 7)\log 3 = \log 24$ Sol: $x = 3; \quad x = 2$

27. $2\log x - \log 16 = \log \frac{x}{2}$ Sol: $x = 0; \quad x = 8$

28. $\log(2x+4) + \log(3x+1) - \log 4 = 2\log(8-x)$ Sol: $x = -42 \quad x = 3$

29. $\frac{\log(35-x^3)}{\log(5-x)} = 3$ Sol: $x = 3 \quad x = 2$

30. $\frac{\log 2 + \log(11-x^2)}{\log(5-x)} = 2$ Sol: $x = \frac{1}{3} \quad x = 3$

31. $\log(5x+4) - \log 2 = \frac{1}{2}\log(x+4)$ Sol: $x = 0$

32. $(x^2 - x + 3)\log 4 = 3\log \frac{1}{4}$ Sol: No tiene solución.

2   Sistemas de Ecuaciones Logarítmicas:

1.
$$\begin{cases} 2\log x - & 5\log y = & -1 \\ 3\log x + & 2\log y = & 8 \end{cases}$$

Sol: $x = 100; \quad y = 10$

2.
$$\begin{cases} 4\log x - 3\log y & = & -1 \\ \log(x \cdot y) & = & 5 \end{cases}$$

Sol: $x = 100; \quad y = 1000$

3.
$$\begin{cases} \log x + \log y^3 & = & 5 \\ \log \frac{x^3}{y^2} & = & 4 \end{cases}$$

Sol: $x = 100; \quad y = 10$

4.
$$\begin{cases} \log(x^2 \cdot y) & = 2 \\ \log \frac{x}{y} & = 1 \end{cases}$$

Sol: $x = 10; \quad y = 1$

5.

$$\begin{cases} \log x^2 - 3\log y &= -1 \\ \log(x \cdot y^2) &= 3 \end{cases}$$

Sol: $x = 10$; $y = 10$

6.

$$\begin{cases} \log x^2 - 3\log y &= 2 \\ \log(\frac{x}{y^2}) &= 3 \end{cases}$$

Sol: $x = 10^{-5}$; $y = 10^{-4}$

7.

$$\begin{cases} \log x - \log y &= 7 \\ \log x + \log y &= 3 \end{cases}$$

Sol: $x = 10^5$; $y = 10^{-2}$

8.

$$\begin{cases} x - y &= 15 \\ \log x + \log y &= 2 \end{cases}$$

Sol: $x = -5$; $y = -20$ o bien $x = 20$; $y = 5$

9.

$$\begin{cases} \log x + 3\log y &= 5 \\ \log \frac{x^2}{y} &= 3 \end{cases}$$

Sol: $x = 100$; $y = 10$

10.

$$\begin{cases} 2\log x^2 &- \log y^2 &= 4 \\ 2\log x &+ \log y^2 &= 2 \end{cases}$$

Sol: $x = 100$; $y = 1$

3 Ecuaciones Exponenciales:

1. $2^{x+1} = 8$ Sol: $x = 2$

2. $2^{x+3} + 4^{x+1} = 320$ Sol: $x = 3$

3. $6^{12-3x} = 216$ Sol: $x = 3$

4. $5^{3x-12} = 125$ Sol: $x = 5$

5. $2^x + 2^{x+3} = 36$ Sol: $x = 2$

6. $3^x + 3^{x-2} = 270$ Sol: $x = 5$

7. $5^x + 5^{x+1} + 5^{x+2} = \frac{31}{25}$ Sol: $x = -2$

8. $5^{2x^2+3x-11} = 125$ Sol: $x = 2$; $x = -\frac{7}{2}$

9. $4^x + 2^{2x-1} = 24$ Sol: $x = 2$; la otra solución no es real.

10. $2^x + 2^{2x} = 6$ Sol: $x = 1$; la otra solución no es real.

11. $3^{x+3} + 9^{x+2} = 4$ Sol: $x = -2$; la otra solución no es real.

12. $4^{2x+1} - 4^{x+2} = 768$ Sol: $x = 2$; la otra solución no es real.

13. $2^x \cdot 3^x = 12 \cdot 18$ Sol: $x = 3$

14. $9^{x+3} = 3^{2x+5}$ Sol: No tiene solución.

15. $8^{x^2+3x+2} = 1$ Sol: $x = -1$; $x = -2$

16. $5^x + 5^{x-1} + x^{x-2} = 31$ Sol: $x = 2$

17. $2^{x+2} = 0,5^{2x-1}$ Sol: $x = -\frac{1}{3}$

18. $\sqrt[3]{a^{7-x}} = a^2$ Sol: $x = 1$

19. $4^x - 5 \cdot 2^x + 4 = 0$ Sol: $x = 2$; $x = 0$

20. $7^{2x+3} - 8 \cdot 7^{x+1} + 1 = 0$ Sol: $x = -1$; $x = -2$

21. $4^x \cdot 5^{x-1} = 1600$ Sol: $x = 3$

22. $10^{x^2-11x+30} = (2 \cdot 5)^2$ Sol: $x = 7$; $x = 4$

23. $3^{x-1} + 3^x + 3^{x+1} = 117$ Sol: $x = 3$

24. $3^{2(x+1)} - 28 \cdot 3^x + 3 = 0$ Sol: $x = -2$; $x = 1$

25. $2^{2x} - 3 \cdot 2^{x+1} + 8 = 0$ Sol: $x = 2$; $x = 1$

26. $\left(\frac{2}{7}\right)^5 = 3,5^{x+1}$ Sol: $x = -6$

27. $5^x - \frac{5}{5^{x-1}} - 24 = 0$ Sol: $x = 2$

28. $\left(4^{3-x}\right)^{2-x} = 1$ Sol: $x = 3$; $x = 2$

29. $2^{1-x^2} = \frac{1}{8}$ Sol: $x = \pm 2$

30. $3^{2x-1} = \sqrt[3]{9^{x^2-\frac{1}{4}}}$ Sol: $x = \frac{11}{2}$; $x = \frac{1}{2}$

31. $3 \cdot 2^{x+3} = 192 \cdot 3^{x-3}$ Sol: No tiene solución.

4   Sistemas de Ecuaciones Exponenciales:

1.

$$\begin{cases} 3 \cdot 2^x - & 4 \cdot 7^y = & -172 \\ 7 \cdot 2^x + & 2 \cdot 7^y = & 154 \end{cases}$$

Sol: $x = 3$; $y = 2$

2.

$$\begin{cases} 4^{x+1} - & 6^y = & 40 \\ 2 \cdot 4^x - & 6^y = & -88 \end{cases}$$

Sol: $x = 3$; $y = 3$

3.

$$\begin{cases} 2 \cdot 3^{x+1} - & 5^{y+2} = & -2639 \\ 4 \cdot 3^x + & 5^y = & 449 \end{cases}$$

Sol: $x = 4$; $y = 4$

4.

$$\begin{cases} 3^x + & 2^y = & 31 \\ 3^{x+1} - & 2^{y+2} = & 65 \end{cases}$$

Sol: $x = 3$; $y = 2$

5.

$$\begin{cases} 5^{x+y} & = & 25^3 \\ 3^{x-y} & = & 25 \end{cases}$$

Sol: $x = 4$; $y = 2$

6.

$$\begin{cases} 15 \cdot & 5^{x-1} & - & 6^y = & 339 \\ 3 \cdot & 5^x & + & 2 \cdot 6^{y+1} = & 807 \end{cases}$$

Sol: $x = 3$; $y = 2$

7.

$$\begin{cases} a^{x+y} & = & a^4 \\ a^{x-y} & = & a^2 \end{cases}$$

Sol: $x = 3$; $y = 1$

8.

$$\begin{cases} 8^y \cdot 2^{2x} = 128 \\ 3^{2y} \cdot 3^{x-1} = 27 \end{cases}$$

Sol: $x = -70; \quad y = 49$

9.

$$\begin{cases} 3^{3x-y} = \sqrt{3^{10}} \\ 3^{2x+y} = 3 \end{cases}$$

Sol: $x = \frac{6}{5}; \quad y = -\frac{7}{5}$

10.

$$\begin{cases} 3 \cdot 2^x - 2 \cdot 3^y = -6 \\ 4 \cdot 2^x - 3 \cdot 3^y = -11 \end{cases}$$

Sol: $x = 2; \quad y = 2$

11.

$$\begin{cases} 3 \cdot 2^x - 5 \cdot 3^y = 3 \\ 2^{x+1} + 3^{y+1} = 59 \end{cases}$$

Sol: $x = 4; \quad y = 2$

12.

$$\begin{cases} 2^x - 3^{y-1} = 5 \\ 2^{x+1} + 8 \cdot 3^y = 712 \end{cases}$$

Sol: $x = 5; \quad y = 4$

13.

$$\begin{cases} 2 \cdot 3^x + 2^{y+3} = 86 \\ 3^x - 2^y = 23 \end{cases}$$

Sol: $x = 3; \quad y = 2$

14.

$$\begin{cases} 2^{x+2y} = 32 \\ 5^{2x-y} = 1 \end{cases}$$

Sol: $x = 1; \quad y = 2$