Examen de Matemáticas 4º de ESO
Diciembre 2010

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2\log(x+3) - 1 = \log(x+5)$$

Solución:

$$\log\left(\frac{(x+3)^2}{10}\right) = \log(x+5) \implies x^2 - 4x - 41 = 0 \implies$$

$$\begin{cases} x = 8,708 \\ x = -4,708 \text{ No Vale} \end{cases}$$

Problema 2

$$3^{2x+1} - 3^{x-1} - 2 = 0$$

Solución:

$$3(3^x)^2 - \frac{3^x}{3} - 2 = 0 \implies 3t^2 - \frac{t}{3} - 2 = 0 \implies \begin{cases} t = 0,874 \\ t = -0,763 \end{cases}$$

$$\begin{cases} t = 0,874 = 3^x \implies x = -0,123 \\ t = -0,763 = 3^x \implies \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^5 y) & = 12 \\ \log\left(\dfrac{x^5}{y}\right) & = 8 \end{cases}$$

Solución:

$$\begin{cases} \log(x^5 y) & = 12 \\ \log\left(\dfrac{x^5}{y}\right) & = 8 \end{cases} \implies \begin{cases} 5u+ & v = & 12 \\ 5u- & v = & 8 \end{cases} \implies$$

$$\begin{cases} u = \log x = 2 \implies x = 100 \\ v = \log y = 2 \implies y = 100 \end{cases}$$

Problema 4

$$\begin{cases} 3^{x-1}- & 5^y & = -1 \\ 3^{x+2}+ & 2\cdot 5^y & = 31 \end{cases}$$

Solución:

$$\begin{cases} \dfrac{3^x}{3} - & 5^y & = -1 \\ 9 \cdot 3^x + & 2 \cdot 5^y & = 31 \end{cases} \implies \begin{cases} \dfrac{u}{3} - & v & = -1 \\ 9u + & 2v & = 31 \end{cases} \implies$$

$$\begin{cases} u = 3 = 3^x \implies x = 1 \\ v = 2 = 5^y \implies y = 0,431 \end{cases}$$

Problema 5

$$\frac{x+1}{2} - \frac{x-3}{7} \leq x - \frac{x-1}{7}$$

Solución:

$$7x + 7 - 2x + 6 \leq 14x - 2x + 2 \implies x \geq \frac{11}{7} \implies \left[\frac{11}{7}, \infty\right)$$

Problema 6

$$\frac{x^2 - 3x + 2}{x + 3} \leq 0$$

Solución:

$$\frac{x^2 - 3x + 2}{x + 3} = \frac{(x-1)(x-2)}{x+3} \leq 0$$

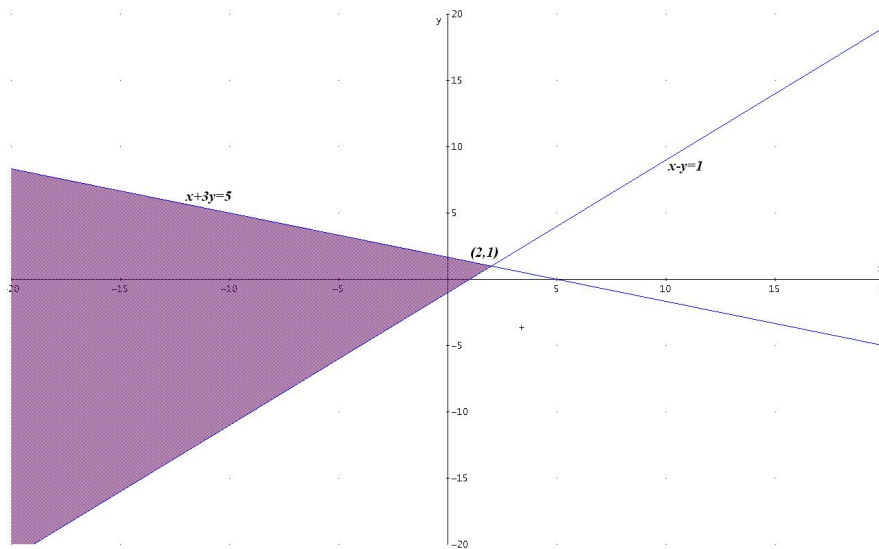La solución es: $(-\infty, -3) \cup [1, 2]$

Problema 7

$$\begin{cases} x + & 3y & \leq 5 \\ x - & y & \leq 1 \end{cases}$$

Solución:

$$x + 3y = 5 \implies \begin{array}{c|c} x & y \\ \hline 0 & 5/3 \\ 5 & 0 \end{array}$$

$$x - y = 1 \implies \begin{array}{c|c} x & y \\ \hline 0 & -1 \\ 1 & 0 \end{array}$$

$$\begin{cases} x + & 3y & = 5 \\ x - & y & = 1 \end{cases} \begin{cases} x = 2 \\ y = 1 \end{cases} \implies (2,1)$$

Problema 8

$$\sqrt{x^2 + 3} = x + 1$$

Solución:

$$(\sqrt{x^2 + 3})^2 = (x + 1)^2 \implies x^2 + 3 = x^2 + 2x + 1$$

$$\implies 2 = 2x \implies x = 1$$

Problema 9

$$\sqrt{x + 4} - \sqrt{x - 1} = 1$$

Solución:

$$\sqrt{x + 4} = 1 + \sqrt{x - 1} \implies x + 4 = 1 + x - 1 + 2\sqrt{x - 1} \implies \sqrt{x - 1} = 2 \implies x = 5$$

Problema 10

$$x^4 - 3x^2 - 4 = 0$$

Solución:

Hacemos $z = x^2 \implies z^2 - 3z - 4 = 0 \implies z = 4 \ y \ z = -1$.

$$z = 4 = x^2 \implies x = \pm 2$$

$$z = -1 = x^2 \ \text{No Vale}$$