Examen de Matemáticas 4º de ESO

Diciembre 2010

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2\log(x-2)-1=\log x$$

Solución:

$$\log\left(\frac{(x-2)^2}{10}\right)=\log x \Longrightarrow x^2-14x+4=0 \Longrightarrow$$

$$\begin{cases} x=13,708 \\ x=0,292 \text{ No Vale} \end{cases}$$

Problema 2

$$2^{2x+1}-2^{x-1}-1=0$$

Solución:

$$2(2^x)^2-\frac{2^x}{2}-1=0 \Longrightarrow 2t^2-\frac{t}{2}-1=0 \Longrightarrow \begin{cases} t=0,843 \\ t=-0,593 \end{cases}$$

$$\begin{cases} t=0,843=2^x \Longrightarrow x=-0,246 \\ t=-0,593=2^x \Longrightarrow \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^3y) & =8 \\ \log\left(\dfrac{x^2}{y}\right) & =2 \end{cases}$$

Solución:

$$\begin{cases} \log(x^3y) & =8 \\ \log\left(\dfrac{x^2}{y}\right) & =2 \end{cases} \Longrightarrow \begin{cases} 3u+ & v= & 8 \\ 2u- & v= & 2 \end{cases} \Longrightarrow$$

$$\begin{cases} u=\log x=2 \Longrightarrow x=100 \\ v=\log y=2 \Longrightarrow y=100 \end{cases}$$

Problema 4

$$\begin{cases} 2^{x+2}- & 3^{y+1} & =5 \\ 2^{x-1}+ & 3^y & =2 \end{cases}$$

Solución:

$$\left\{\begin{array}{rrl} 4\cdot 2^x - 3\cdot 3^y & 5 & \\ \frac{2^x}{2}+ & \cdot 3^y & = 2 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} 4u- & 3v & = 5 \\ \frac{u}{2}+ & v & = 2 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 2 = 2^x \Longrightarrow x = 1 \\ v = 1 = 3^y \Longrightarrow y = 0 \end{array}\right.$$

Problema 5

$$\frac{x-1}{3} + 1 \geq \frac{x-1}{4} - \frac{2x+1}{6}$$

Solución:

$$4x - 4 + 12 \geq 3x - 3 - 4x - 2 \Longrightarrow x \geq -\frac{13}{5} \Longrightarrow \left[-\frac{13}{5}, \infty\right)$$

Problema 6

$$\frac{x^2 - x - 2}{x - 3} \leq 0$$

Solución:

$$\frac{x^2 - x - 2}{x - 3} = \frac{(x+1)(x-2)}{x-3} \geq 0$$

La solución es: $(-\infty, -1] \cup [2, 3)$

Problema 7

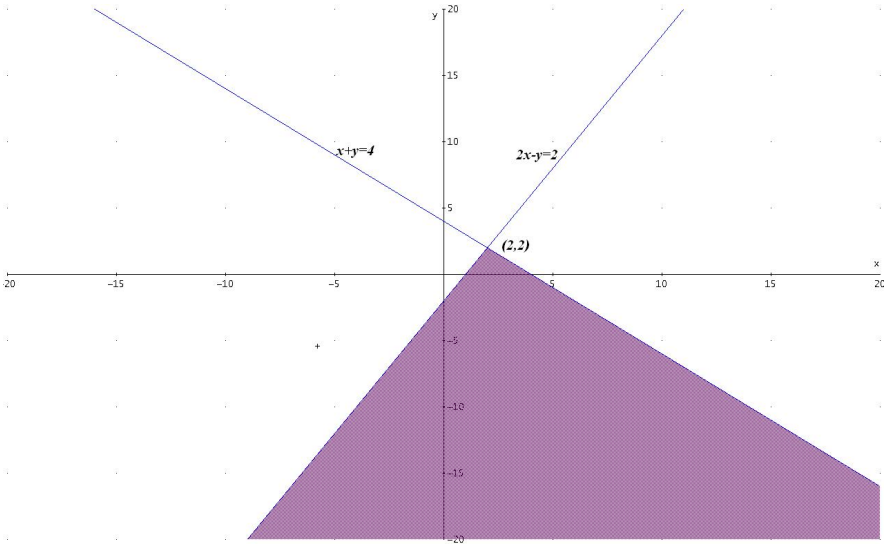
$$\left\{\begin{array}{rrl} 2x- & y & \geq 2 \\ x+ & y & \leq 4 \end{array}\right.$$

Solución:

$$2x - y = 2 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -2 \\ 1 & 0 \end{array}$$

$$x + y = 4 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & 4 \\ 4 & 0 \end{array}$$

$$\left\{\begin{array}{rrl} 2x- & y & = 2 \\ x+ & y & = 4 \end{array}\right. \left\{\begin{array}{l} x = 2 \\ y = 2 \end{array}\right. \Longrightarrow (2, 2)$$



Problema 8

$$\sqrt{3x^2+1} = x+1$$

Solución:

$$(\sqrt{3x^2+1})^2 = (x+1)^2 \Longrightarrow 3x^2+1 = x^2+2x+1$$

$$\Longrightarrow 2x^2-2x=0 \Longrightarrow \ x=0, \ \text{ y } \ x=1$$

Problema 9

$$\sqrt{x-3} - \sqrt{x} = -1$$

Solución:

$$\sqrt{x-3} = \sqrt{x}-1 \Longrightarrow x-3 = x+1-2\sqrt{x} \Longrightarrow \sqrt{x}=2 \Longrightarrow \ x=4$$

Problema 10

$$x^4+x^2-2=0$$

Solución:

Hacemos $z = x^2 \Longrightarrow z^2+z-2=0 \Longrightarrow \ z=1 \ y \ z=-2.$

$$z=1=x^2 \Longrightarrow x=\pm 1$$

$$z=-2=x^2 \text{ No Vale}$$