Examen de Matemáticas 4º de ESO
Noviembre 2007

Resolver las siguientes ecuaciones y sistemas:

Problema 1 Resolver:

1. $\log(x + 3) + \log x = 1$

2. $\log(x - 5) - 2 = 2 \log x$

Solución:

1. $\log(x + 3) + \log x = 1 \Longrightarrow x^2 + 3x = 10 \Longrightarrow x = 2$ y $x = -5$ que no vale.

2. $\log(x - 5) - 2 = 2 \log x \Longrightarrow x - 5 = 100x^2 \Longrightarrow$ No tiene solución.

Problema 2

$$2^{2x-1} - 2^{x-1} - 4 = 0$$

Solución:

$$\frac{t^2}{2} - \frac{t}{2} - 4 = 0 \Longrightarrow t^2 - t - 8 = 0$$

$$\begin{cases} t = 2^x = 3,372281323 \Longrightarrow x = 1,754 \\ t = 2^x = -2,372281323 \text{ No Vale} \end{cases}$$

Problema 3

$$\begin{cases} \log(x^2 y) & = 8 \\ \log\left(\frac{x}{y^2}\right) & = 4 \end{cases}$$

Solución:

$$\begin{cases} 2\log x+ & \log y = 8 \\ \log x- & 2\log y = 4 \end{cases} \Longrightarrow \begin{cases} 2u+ & v = 8 \\ u- & 2v = 4 \end{cases} \Longrightarrow$$

$$\begin{cases} u = \log x = 4 \Longrightarrow x = 10000 \\ v = \log y = 0 \Longrightarrow y = 1 \end{cases}$$

Problema 4

$$\begin{cases} 3 \cdot 5^x- & 3^y & = 2 \\ 5^x+ & 3^y & = 6 \end{cases}$$

Solución:

$$\begin{cases} 3 \cdot 5^x- & 3^y & = 2 \\ 5^x+ & 3^y & = 6 \end{cases} \Longrightarrow \begin{cases} 3u- & v & = 2 \\ \\ u+ & v & = 6 \end{cases} \Longrightarrow$$

$$\begin{cases} u = 2 = 5^x \Longrightarrow x = 0,431 \\[2mm] v = 4 = 3^y \Longrightarrow y = 1,2619 \end{cases}$$

Problema 5

$$\frac{x-1}{5} - 1 \geq \frac{x}{2} - \frac{x+1}{5}$$

Solución:

$$2x - 2 - 10 \geq 5x - 2x - 2 \Longrightarrow -x \geq 10 \Longrightarrow x \leq -10 \Longrightarrow (-\infty, -10]$$

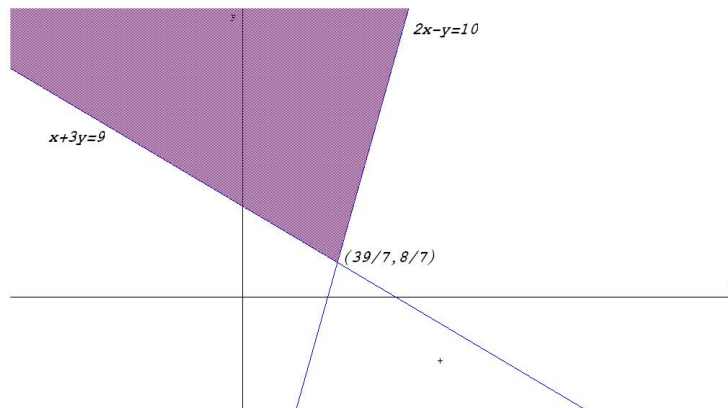**Problema 6**

$$\frac{x^2 - 6x + 5}{x+3} \leq 0$$

Solución:

$$\frac{x^2 - 6x + 5}{x+3} = \frac{(x-5)(x-1)}{x-3} \leq 0$$

La solución es: $(-\infty, -3) \cup [1, 5]$

Problema 7

$$\begin{cases} 2x - \quad y \quad \leq 10 \\ x + \quad 3y \quad \geq 9 \end{cases}$$

Solución:

$$2x - y = 10 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & -10 \\ 5 & 0 \end{array}$$

$$x + 3y = 9 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & 3 \\ 9 & 0 \end{array}$$

$$\begin{cases} 2x - \quad y \quad = 10 \\ x + \quad 3y \quad = 9 \end{cases} \begin{cases} x = 39/7 \\ y = 8/7 \end{cases} \Longrightarrow \left(\frac{39}{7}, \frac{8}{7} \right)$$

Problema 8

$$\sqrt{5x-1} - \sqrt{x+2} = 1$$

Solución:

$$\sqrt{5x-1} = 1+\sqrt{x+2} \implies 5x-1 = 1+(x+2)+2\sqrt{x+2} \implies 2x-2 = \sqrt{x+2}$$

$$4x^2 - 8x + 4 = x + 2 \implies 4x^2 - 9x + 2 = 0$$

$$\begin{cases} x = 1/4 \ \ \text{No Vale} \\ x = 2 \end{cases}$$

Problema 9

$$\sqrt{3x+1} + x = 3$$

Solución:

$$3x + 1 = 9 + x^2 - 6x \implies x^2 - 9x + 8 = 0$$

$$\begin{cases} x = 8 \ \ \text{No Vale} \\ x = 1 \end{cases}$$

Problema 10

$$x^4 - 6x^2 - 27 = 0$$

Solución:

Hacemos $z = x^2 \implies z^2 - 6z - 27 = 0 \implies z = 9 \ y \ z = -3.$

$$z = 9 = x^2 \implies x = \pm 3$$

$$z = -3 = x^2 \ \text{No Vale}$$