# Examen de Matemáticas 4º de ESO
## Diciembre 2004

Resolver las siguientes ecuaciones y sistemas:

### Problema 1

$$\log x - \log(1 - x) = 2$$

**Solución:**

$$\log\left(\frac{x}{1-x}\right) = \log 100 \Longrightarrow 101x = 100 \Longrightarrow x = \frac{100}{101}$$

### Problema 2

$$2^x - 2^{x+1} + 1 = 0$$

**Solución:**

$$2^x - 2 \cdot 2^x + 1 = 0 \Longrightarrow t - 2t + 1 = 0 \Longrightarrow t = 1$$

$$t = 2^x = 1 \Longrightarrow x = 0$$

### Problema 3

$$\left\{ \begin{array}{ll} \log x + & \log y = 3 \\ 2\log x - & \log y = 0 \end{array} \right.$$

**Solución:**

$$\left\{ \begin{array}{ll} \log x + & \log y = 3 \\ 2\log x - & \log y = 0 \end{array} \right. \Longrightarrow \left\{ \begin{array}{lll} u + & v = & 3 \\ 2u - & v = & 0 \end{array} \right. \Longrightarrow$$

$$\left\{ \begin{array}{l} u = \log x = 1 \Longrightarrow x = 10 \\ v = \log y = 2 \Longrightarrow y = 100 \end{array} \right.$$

### Problema 4

$$\left\{ \begin{array}{lll} 2^x - & 3^y & = 1 \\ 2^x + & 3^y & = 3 \end{array} \right.$$

**Solución:**

$$\left\{ \begin{array}{lll} 2^x - & 3^y & = 1 \\ 2^x + & 3^y & = 3 \end{array} \right. \Longrightarrow \left\{ \begin{array}{lll} u - & v & = 1 \\ \\ u + & v & = 3 \end{array} \right. \Longrightarrow$$

$$\left\{ \begin{array}{l} u = 2 = 2^x \Longrightarrow x = 1 \\ \\ v = 1 = 3^y \Longrightarrow y = 0 \end{array} \right.$$

**Problema 5**

$$\frac{x}{2} + \frac{x-1}{6} < 1 - \frac{x+1}{3}$$

**Solución:**

$$3x + x - 1 < 6 - 2x - 2 \Longrightarrow x < \frac{5}{6} \Longrightarrow \left(-\infty, \frac{5}{6}\right)$$

**Problema 6**

$$x^2 - x - 2 < 0$$
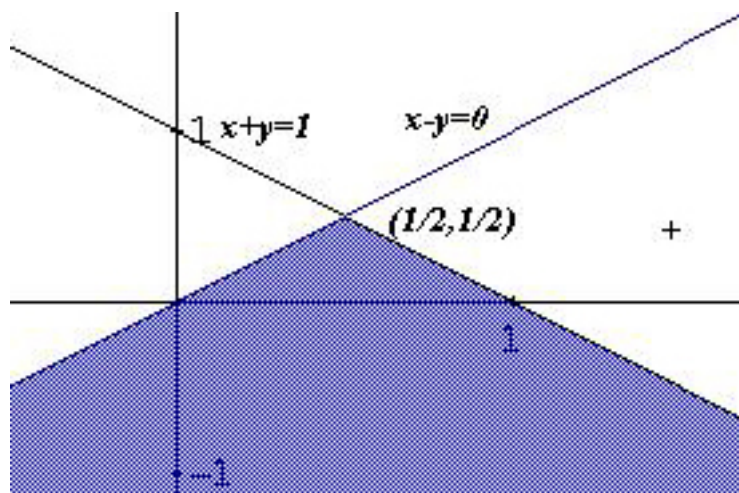
**Solución:**

$$x^2 - x - 2 = (x-2)(x+1) < 0$$

|             | $(-\infty, -1)$ | $(-1, 2)$ | $(2, \infty)$ |
|:-----------:|:---------------:|:---------:|:-------------:|
| $x + 1$     | $-$             | $+$       | $+$           |
| $x - 2$     | $-$             | $-$       | $+$           |
| $x^2 - x - 2$ | $+$           | $-$       | $+$           |

La solución es: $(-1, 2)$

**Problema 7**

$$\begin{cases} x+ \quad y \quad < 1 \\ x- \quad y \quad > 0 \end{cases}$$

**Solución:**

$$x + y = 1 \implies \begin{array}{c|c} x & y \\ \hline 0 & 1 \\ 1 & 0 \end{array}$$

$$x - y = 0 \implies \begin{array}{c|c} x & y \\ \hline 0 & 0 \\ 1 & 1 \end{array}$$

$$\begin{cases} x+ \ y \ = 1 \\ x- \ y \ = 0 \end{cases} \begin{cases} x = 1/2 \\ y = 1/2 \end{cases} \implies (1/2, 1/2)$$

**Problema 8**

$$\sqrt{2x - 1} + x = 8$$

**Solución:**

$$2x - 1 = 64 + x^2 - 16x \implies x^2 - 18x + 65 = 0 \implies$$

$$\begin{cases} x = 5 \\ x = 13 \ \text{No Vale} \end{cases}$$

**Problema 9**

$$\sqrt{x + 1} = x - 1$$

**Solución:**

$$x + 1 = 1 + x^2 - 2x \implies x^2 - 3x = 0 \implies x(x - 3) = 0$$

$$\begin{cases} x = 3 \\ x = 0 \ \text{No Vale} \end{cases}$$

**Problema 10**

$$x^4 - 2x^2 - 8 = 0$$

**Solución:**

Hacemos $z = x^2 \implies z^2 - 2z - 8 = 0 \implies z = 4 \ y \ z = -2.$

$$z = 4 = x^2 \implies x = \pm 2$$

$$z = -1 = x^2 \ \text{No Vale}$$