Examen de Matemáticas 4º de ESO
Noviembre 2004

Resolver las siguientes ecuaciones y sistemas:

Problema 1

$$2\log(x+1) - \log x = 1$$

Solución:

$$\log\left(\frac{(x+1)^2}{x}\right) = \log 10 \implies x^2 - 8x + 1 = 0 \implies$$

$$\begin{cases} x = 0,127 \\ x = 7,873 \end{cases}$$

Problema 2

$$3^{2x+1} - 3^{x-1} - 1 = 0$$

Solución:

$$3(3^x)^2 - \frac{3^x}{3} - 1 = 0 \implies 3t^2 - \frac{t}{3} - 1 = 0 \implies \begin{cases} t = 0,63557 \\ t = -0,524461 \end{cases}$$

$$\begin{cases} t = 0,63557 = 3^x \implies x = -0,41255 \\ t = -0,524461 = 3^x \implies \text{No Vale} \end{cases}$$

Problema 3

$$\begin{cases} 2\log(xy) = 3 \\ \log\left(\dfrac{x}{y^2}\right) = 5 \end{cases}$$

Solución:

$$\begin{cases} 2\log x + & 2\log y = & 3 \\ \log x - & 2\log y = & 5 \end{cases} \implies \begin{cases} 2u + & 2v = & 3 \\ u - & 2v = & 5 \end{cases} \implies$$

$$\begin{cases} u = \log x = 8/3 \implies x = 464,1588 \\ v = \log y = -7/6 \implies y = 0,068129 \end{cases}$$

Problema 4

$$\begin{cases} 3^{x-2} + & 2^y & = 1 \\ 2^x + & 3 \cdot 2^y & = 5 \end{cases}$$

Solución:

$$\begin{cases} \dfrac{3^x}{9} + & 2^y & = 1 \\ 3^x + & 3 \cdot 2^y & = 5 \end{cases} \Longrightarrow \begin{cases} \dfrac{u}{9} + & v & = 1 \\ u + & 3v & = 5 \end{cases} \Longrightarrow$$

$$\begin{cases} u = 3 = 3^x \Longrightarrow x = 1 \\ v = \dfrac{2}{3} = 2^y \Longrightarrow y = -0,585 \end{cases}$$

Problema 5

$$\frac{x-1}{5} - \frac{x}{15} \leq \frac{x+1}{3} + 2$$

Solución:

$$3x - 3 + x \leq 30 + 5x + 5 \Longrightarrow x \geq -38 \Longrightarrow [-38, \infty)$$

Problema 6

$$\frac{x^2 + 3x - 40}{x - 2} \geq 0$$

Solución:

$$\frac{x^2 + 3x - 40}{x - 2} = \frac{(x+8)(x-5)}{x-2} \geq 0$$

	$(-\infty, -8)$	$(-8, 2)$	$(2, 5)$	$(5, \infty)$
$x + 8$	$-$	$+$	$+$	$+$
$x - 2$	$-$	$-$	$+$	$+$
$x - 5$	$-$	$-$	$-$	$+$
$\frac{x^2+3x-40}{x-2}$	$-$	$+$	$-$	$+$

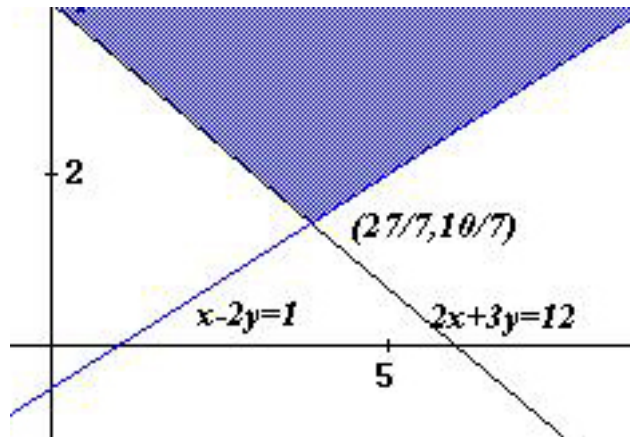La solución es: $[-8, 2) \cup [5, \infty)$

Problema 7

$$\begin{cases} 2x + & 3y & \geq 12 \\ x - & 2y & < 1 \end{cases}$$

Solución:

$$2x + 3y = 12 \Longrightarrow \begin{array}{c|c} x & y \\ \hline 0 & 4 \\ 6 & 0 \end{array}$$

$$x - 2y = 1 \implies \begin{array}{c|c} x & y \\ \hline 0 & -1/2 \\ 1 & 0 \end{array}$$

$$\begin{cases} 2x+ & 3y & = 12 \\ x- & 2y & = 1 \end{cases} \begin{cases} x = 27/7 \\ y = 10/7 \end{cases} \implies (27/7, 10/7)$$

Problema 8

$$\sqrt{x-3} + \sqrt{x} = 4$$

Solución:

$$\sqrt{x-3} = 4 - \sqrt{x} \implies x - 3 = 16 + x - 8\sqrt{x} \implies -19 = -8\sqrt{x} \implies x = \frac{361}{64}$$

Problema 9

$$\sqrt{x+4} = x - 1$$

Solución:

$$x + 4 = x^2 + 1 - 2x \implies x^2 - 3x - 3 = 0 \implies$$

$$\begin{cases} x = 3,7912 \\ \\ x = -0,79128 \ \text{No Vale} \end{cases}$$

Problema 10

$$x^4 - 80x^2 - 81 = 0$$

Solución:

Hacemos $z = x^2 \implies z^2 - 80z - 81 = 0 \implies z = 81 \ y \ z = -1.$

$$z = 81 = x^2 \implies x = \pm 9$$

$$z = -1 = x^2 \ \text{No Vale}$$