Examen de Matemáticas 2º de Bachillerato CN
Diciembre 2022

Problema 1 Calcule el límite: $\lim_{x\longrightarrow 0}\left(\dfrac{2}{\ln((1+x)^2)}-\dfrac{1}{x}\right)$

Solución:

$$\lim_{x\longrightarrow 0}\left(\frac{2}{\ln((1+x)^2)}-\frac{1}{x}\right)=\lim_{x\longrightarrow 0}\frac{2x-\ln((1+x)^2)}{x\ln((1+x)^2)}=\left[\frac{0}{0}\right]=$$

$$\lim_{x\longrightarrow 0}\frac{2-\frac{2(1+x)}{(1+x)^2}}{\ln((1+x)^2)+x\frac{2(1+x)}{(1+x)^2}}=\lim_{x\longrightarrow 0}\frac{\frac{2x}{1+x}}{\ln((1+x)^2)+\frac{2x}{1+x}}=$$

$$\lim_{x\longrightarrow 0}\frac{2x}{(1+x)\ln((1+x)^2)+2x}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0}\frac{2}{\ln((1+x)^2)+(1+x)\frac{2(1+x)}{(1+x)^2}+2}=$$

$$\lim_{x\longrightarrow 0}\frac{2}{\ln((1+x)^2)+4}=\frac{1}{2}$$

Problema 2 Sea $f(x)$ la función definida en $(0,\infty)$ dada por $f(x)=x\ln(x)$, donde ln denota el logaritmo neperiano.

a) Calcule $\lim_{x\longrightarrow 0^+} f(x)$.

b) Calcule $\int_2^e f(x)\,dx$.

Solución:

a) $\lim_{x\longrightarrow 0^+} x\ln(x)=[0\cdot(-\infty)]=\lim_{x\longrightarrow 0^+}\dfrac{\ln x}{\frac{1}{x}}=\left[\dfrac{-\infty}{+\infty}\right]=\lim_{x\longrightarrow 0^+}\dfrac{\frac{1}{x}}{-\frac{1}{x^2}}=\lim_{x\longrightarrow 0^+}(-x)=0$

b) $F(x)=\int f(x)\,dx=\left[\begin{array}{l} u=\ln x\Longrightarrow du=\frac{1}{x}dx \\ dv=xdx\Longrightarrow v=\frac{x^2}{2}\end{array}\right]=\dfrac{x^2\ln x}{2}-\dfrac{1}{2}\int x\,dx=\dfrac{x^2\ln x}{2}-\dfrac{x^2}{4}=\dfrac{x^2(2\ln x-1)}{4}$

$\int_2^e f(x)\,dx=F(e)-F(2)=\dfrac{e^2}{4}+1-2\ln 2\simeq 1,461$

Problema 3 Calcula razonadamente los siguientes límites:

a) $\lim_{x\longrightarrow 0}\dfrac{\cos x-1}{x\sin x}$

b) $\lim_{x \longrightarrow 0} \dfrac{x \sin x}{3 \cos x - 3}$

Solución:

a) $\lim_{x \longrightarrow 0} \dfrac{\cos x - 1}{x \sin x} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow 0} \dfrac{-\sin x}{\sin x + x \cos x} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow 0} \dfrac{-\cos x}{\cos x + \cos x - x \sin x} = -\dfrac{1}{2}$

b) $\lim_{x \longrightarrow 0} \dfrac{x \sin x}{3 \cos x - 3} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow 0} \dfrac{\sin x + x \cos x}{-3 \sin x} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow 0} \dfrac{\cos x + \cos x - x \sin x}{-3 \cos x} = -\dfrac{2}{3}$

Problema 4 Calcula razonadamente los siguientes límites:

a) $\lim_{x \longrightarrow 1} \left(\dfrac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}}$

b) $\lim_{x \longrightarrow -1} \dfrac{-e^{x^2-1} - x}{x^2 + 4x + 3}$

Solución:

a) $L = \lim_{x \longrightarrow 1} \left(\dfrac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} = [1^{\infty}] = e^{\lambda}$

$\lambda = \lim_{x \longrightarrow 1} \dfrac{x}{x-1} \left(\dfrac{2e^{x-1}}{x+1} - 1\right) = \lim_{x \longrightarrow 1} \dfrac{2xe^{x-1} - x^2 - x}{x^2 - 1} = \left[\dfrac{0}{0}\right] =$
$\lim_{x \longrightarrow 1} \dfrac{2e^{x-1} + 2xe^{x-1} - 2x - 1}{2x} = \dfrac{1}{2} \Longrightarrow L = e^{1/2}$

b) $\lim_{x \longrightarrow -1} \dfrac{-e^{x^2-1} - x}{x^2 + 4x + 3} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow -1} \dfrac{-2xe^{x^2-1} - 1}{2x + 4} = \dfrac{1}{2}$

Problema 5 Se pide:

a) Calcule $\lim_{x \longrightarrow 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$

b) Calcule $\lim_{x \longrightarrow +\infty} \dfrac{\sqrt{x}}{1 + x}$

Solución:

a) $\lim_{x \longrightarrow 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right) = \lim_{x \longrightarrow 0} \dfrac{e^x - 1 - x}{x(e^x - 1)} = \left[\dfrac{0}{0}\right] = \lim_{x \longrightarrow 0} \dfrac{e^x - 1}{e^x - 1 + xe^x} = \left[\dfrac{0}{0}\right] =$
$\lim_{x \longrightarrow 0} \dfrac{e^x}{e^x + e^x + xe^x} = \dfrac{1}{2}$

b) $\lim_{x\longrightarrow +\infty} \frac{\sqrt{x}}{1+x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\longrightarrow +\infty} \frac{\frac{1}{2\sqrt{x}}}{1} = 0$

Problema 6 Considérese la función $f(x) = x^2 e^{-x}$, se pide:

a) Calcular los límites $\lim_{x\longrightarrow +\infty} f(x)$ y $\lim_{x\longrightarrow -\infty} f(x)$

b) Calcular $\int f(x)\,dx$.

Solución:

a) $\lim_{x\longrightarrow +\infty} f(x) = \lim_{x\longrightarrow +\infty} \frac{x^2}{e^x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\longrightarrow +\infty} \frac{2x}{e^x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\longrightarrow +\infty} \frac{2}{e^x} = 0$

$\lim_{x\longrightarrow -\infty} f(x) = \lim_{x\longrightarrow -\infty} (x^2 e^{-x}) = \infty$

b)

$$\int x^2 e^{-x}\,dx = \left[\begin{array}{l} u = x^2 \Longrightarrow du = 2xdx \\ dv = e^{-x}dx \Longrightarrow v = -e^{-x} \end{array}\right] = -x^2e^{-x} + 2\int xe^{-x}\,dx =$$

$$\left[\begin{array}{l} u = x \Longrightarrow du = dx \\ dv = e^{-x}dx \Longrightarrow v = -e^{-x} \end{array}\right] = -x^2e^{-x} + 2\left[-xe^{-x} + \int e^{-x}\,dx\right] =$$

$$-x^2e^{-x}+2\left[-xe^{-x} - e^{-x}\right]+C = -x^2e^{-x}-2xe^{-x}-2e^{-x}+C = -e^{-x}(x^2+2x+2)+C$$