Examen de Matemáticas 2ºBachillerato(CN)
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Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & -3 & 2 \\ 2 & m & -3 \\ m & m & 0 \end{pmatrix}$$

a) Calcular los valores de m para los que la matriz A es inversible.

b) Calcular A^{-1} para $m = 1$.

Solución:

a)

$$\begin{vmatrix} m & -3 & 2 \\ 2 & m & -3 \\ m & m & 0 \end{vmatrix} = m(m+13) = 0 \Longrightarrow m = 0, \quad m = -13$$

Si $m = -13$ o $m = 0 \Longrightarrow |A| = 0 \Longrightarrow \nexists A^{-1}$.

Si $m \neq -13$ y $m \neq 0 \Longrightarrow |A| \neq 0 \Longrightarrow \exists A^{-1}$.

b) $A = \begin{pmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 1 & 1 & 0 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} 3/14 & 1/7 & 1/2 \\ -3/14 & -1/7 & 1/2 \\ 1/14 & -2/7 & 1/2 \end{pmatrix}$

Problema 2 Resolver la ecuación matricial $AX - BX = 3I - CX$. Donde

$$A = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}; \quad B = \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix}$$

Solución:

$$AX - BX = 3I - CX \Longrightarrow X = 3(A - B + C)^{-1}$$

$$A - B + C = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -5 & -3 \end{pmatrix}$$

$$(A + B - C)^{-1} = \begin{pmatrix} 3/5 & -1/5 \\ -1 & 0 \end{pmatrix}$$

$$X = 3(A - B + C)^{-1} = \begin{pmatrix} 9/5 & -3/5 \\ -3 & 0 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} x & x & 1 & 0 \\ 1 & x & 0 & x \\ 0 & 1 & x & x \\ x & 0 & x & 1 \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & x & 1 & 0 \\ 1 & x & 0 & x \\ 0 & 1 & x & x \\ x & 0 & x & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 + \mathrm{F}_2 + \mathrm{F}_3 + \mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} 2x+1 & 2x+1 & 2x+1 & 2x+1 \\ 1 & x & 0 & x \\ 0 & 1 & x & x \\ x & 0 & x & 1 \end{vmatrix} =$$

$$(2x+1)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 0 & x \\ 0 & 1 & x & x \\ x & 0 & x & 1 \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2 - \mathrm{C}_1 \\ \mathrm{C}_3 - \mathrm{C}_1 \\ \mathrm{C}_4 - \mathrm{C}_1 \end{bmatrix} = (2x+1)\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & -1 & x-1 \\ 0 & 1 & x & x \\ x & -x & 0 & 1-x \end{vmatrix} =$$

$$(2x+1)\begin{vmatrix} x-1 & -1 & x-1 \\ 1 & x & x \\ -x & 0 & 1-x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1 + \mathrm{F}_3 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \end{bmatrix} = (2x+1)\begin{vmatrix} -1 & -1 & 0 \\ 1 & x & x \\ -x & 0 & 1-x \end{vmatrix} =$$

$$\begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2 - \mathrm{C}_1 \\ \mathrm{C}_3 \end{bmatrix} = -(2x+1)\begin{vmatrix} 1 & 0 & 0 \\ 1 & x-1 & x \\ -x & x & 1-x \end{vmatrix} = -(2x+1)\begin{vmatrix} x-1 & x \\ x & 1-x \end{vmatrix} =$$

$$-(2x+1)[-(x-1)^2 - x^2] = (2x+1)(2x^2-2x+1) = 4x^3 - 2x^2 + 1$$