Examen de Matemáticas 2º de Bachillerato CN
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Problema 1 Calcule el límite: $\lim\limits_{x\longrightarrow+\infty}\dfrac{\ln(1+x^2)}{\ln(1+e^x)}$

Solución:

$$\lim_{x\longrightarrow+\infty}\frac{\ln(1+x^2)}{\ln(1+e^x)}=\left[\frac{\infty}{\infty}\right]=\lim_{x\longrightarrow+\infty}\frac{\frac{2x}{1+x^2}}{\frac{e^x}{1+e^x}}=\lim_{x\longrightarrow+\infty}\frac{2x(1+e^x)}{e^x(1+x^2)}=\lim_{x\longrightarrow+\infty}\frac{2x+2xe^x}{e^x+x^2e^x}=\left[\frac{\infty}{\infty}\right]=$$

$$\lim_{x\longrightarrow+\infty}\frac{2+2e^x+2xe^x}{e^x+2xe^x+x^2e^x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\longrightarrow+\infty}\frac{2e^x+2e^x+2xe^x}{e^x+2e^x+2xe^x+2xe^x+x^2e^x}=$$

$$\lim_{x\longrightarrow+\infty}\frac{e^x(4+2x)}{e^x(3+4x+x^2)}=\lim_{x\longrightarrow+\infty}\frac{4+2x}{3+4x+x^2}=\left[\frac{\infty}{\infty}\right]=\lim_{x\longrightarrow+\infty}\frac{2x}{x^2}=\lim_{x\longrightarrow+\infty}\frac{2}{x}=0$$

Problema 2 Calcule el límite: $\lim\limits_{x\longrightarrow 0^+}(1+x-\sin x)^{1/x^3}$

Solución:

$$\lim_{x\longrightarrow 0^+}(1+x-\sin x)^{1/x^3}=[1^\infty]=e^\lambda$$

$$\lambda=\lim_{x\longrightarrow 0^+}\frac{1}{x^3}(1+x-\sin x-1)=\lim_{x\longrightarrow 0^+}\frac{x-\sin x}{x^3}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0^+}\frac{1-\cos x}{3x^2}=\left[\frac{0}{0}\right]=$$

$$\lim_{x\longrightarrow 0^+}\frac{\sin x}{6x}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0^+}\frac{\cos x}{6}=\frac{1}{6}$$

Luego

$$\lim_{x\longrightarrow 0^+}(1+x-\sin x)^{1/x^3}=e^{1/6}$$

Problema 3 Calcule el límite: $\lim\limits_{x\longrightarrow 0}(1+x)^{2/\tan x}$

Solución:

$$\lim_{x\longrightarrow 0}(1+x)^{2/\tan x}=[1^\infty]=e^\lambda$$

$$\lambda=\lim_{x\longrightarrow 0^+}\frac{2}{\tan x}(1+x-1)=\lim_{x\longrightarrow 0}\frac{2x}{\tan x}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0^+}\frac{2}{1/\cos^2 x}=2$$

Luego

$$\lim_{x\longrightarrow 0}(1+x)^{2/\tan x}=e^2$$

Problema 4 Calcula razonadamente los siguientes límites:

a) $\lim\limits_{x\longrightarrow 1}\left(\dfrac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}}$

b) $\lim_{x\longrightarrow -1} \dfrac{-e^{x^2-1}-x}{x^2+4x+3}$

Solución:

a) $L = \lim_{x\longrightarrow 1} \left(\dfrac{2e^{x-1}}{x+1}\right)^{\frac{x}{x-1}} = [1^\infty] = e^\lambda$

$\lambda = \lim_{x\longrightarrow 1} \dfrac{x}{x-1}\left(\dfrac{2e^{x-1}}{x+1}-1\right) = \lim_{x\longrightarrow 1} \dfrac{2xe^{x-1}-x^2-x}{x^2-1} = \left[\dfrac{0}{0}\right] =$

$\lim_{x\longrightarrow 1} \dfrac{2e^{x-1}+2xe^{x-1}-2x-1}{2x} = \dfrac{1}{2} \Longrightarrow L = e^{1/2}$

b) $\lim_{x\longrightarrow -1} \dfrac{-e^{x^2-1}-x}{x^2+4x+3} = \left[\dfrac{0}{0}\right] = \lim_{x\longrightarrow -1} \dfrac{-2xe^{x^2-1}-1}{2x+4} = \dfrac{1}{2}$

Problema 5 Se pide:

a) Calcular $\lim_{x\longrightarrow 0} \dfrac{e^x-\cos x}{\ln(x+1)}$

b) Calcular $\displaystyle\int \dfrac{(\ln x)^2}{x}\,dx$

Solución:

a) $\lim_{x\longrightarrow 0} \dfrac{e^x-\cos x}{\ln(x+1)} = \left[\dfrac{0}{0}\right] = \lim_{x\longrightarrow 0} \dfrac{e^x+\sin x}{1/(x+1)} = \lim_{x\longrightarrow 0}[(x+1)(e^x+\sin x)] = 1$

b) $\displaystyle\int \dfrac{(\ln x)^2}{x}\,dx = \begin{bmatrix} t=\ln x \\ dt = \dfrac{1}{x}dx \\ dx = xdt \end{bmatrix} = \int \dfrac{t^2}{x}\,xdt = \int t^2\,dt = \dfrac{t^3}{3}+C = \dfrac{(\ln x)^3}{3}+C$

Problema 6 Se pide:

a) Calcular $\lim_{x\longrightarrow 0} \dfrac{e^x-\cos x - x}{e^x+\sin x-1}$

b) Calcular $\displaystyle\int_0^{\pi/2} (\sin x+\cos x)\,dx$

Solución:

a) $\lim_{x\longrightarrow 0} \dfrac{e^x-\cos x-x}{e^x+\sin x-1} = \left[\dfrac{0}{0}\right] = \lim_{x\longrightarrow 0} \dfrac{e^x+\sin x-1}{e^x+\cos x} = 0$

b) $\displaystyle\int_0^{\pi/2} (\sin x+\cos x)\,dx = -\cos x+\sin x]_0^{\pi/2} = 0+1-(-1+0) = 2$

Problema 7 Se pide:

a) Calcular $\displaystyle\lim_{x\longrightarrow 1}\frac{\sqrt{x^2-x+1}-\sqrt{2x-1}}{1-x}$

b) Dada la función $f(x)=\dfrac{2x-e^{-x}}{x^2+e^{-x}}$, hallar la función primitiva suya $F(x)$ que verifique $F(0)=3$.

Solución:

a) $$\lim_{x\longrightarrow 1}\frac{\sqrt{x^2-x+1}-\sqrt{2x-1}}{1-x}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 1}\frac{\frac{2x-1}{2\sqrt{x^2-x+1}}-\frac{2}{2\sqrt{2x-1}}}{-1}=\frac{1}{2}$$

b) $$F(x)=\int\frac{2x-e^{-x}}{x^2+e^{-x}}\,dx=\left[\begin{array}{l} t=x^2+e^{-x}\\ dt=(2x-e^{-x})dx\\ dx=\dfrac{dt}{2x-e^{-x}}\end{array}\right]=\int\frac{2x-e^{-x}}{t}\cdot\frac{dt}{2x-e^{-x}}=$$
$$\int\frac{1}{t}\,dt=\ln|t|+C=\ln|x^2+e^{-x}|+C$$
$$F(0)=\ln 1+C=C=3\Longrightarrow F(x)=\ln|x^2+e^{-x}|+3$$

Problema 8 Calcule $\displaystyle\lim_{x\longrightarrow 0}\frac{\cos^2 x-1}{1+2x-e^{2x}}$.

Solución:

$$\lim_{x\longrightarrow 0}\frac{\cos^2 x-1}{1+2x-e^{2x}}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0}\frac{-2\sin x\cos x}{2-2e^{2x}}=\lim_{x\longrightarrow 0}\frac{-\sin 2x}{2(1-e^{2x})}=\left[\frac{0}{0}\right]=$$
$$\lim_{x\longrightarrow 0}\frac{-2\cos 2x}{-4e^{2x}}=\frac{-2}{-4}=\frac{1}{2}$$

Problema 9 Se pide:

a) Calcular el siguiente límite: $\displaystyle\lim_{x\longrightarrow 0}\left(\frac{1-\sin x\cos x}{1+\sin x\cos x}\right)^{\frac{1}{\sin x}}$

b) Determinar el valor de la constante real a que se satisfaga la siguiente igualdad:

$$\lim_{x\longrightarrow 4}\frac{\tan\left(\left(\frac{\pi}{8}+1\right)\sqrt{x}-2\right)}{x^2-16+ax}=\frac{1}{32}$$

Solución:

a) $$L=\lim_{x\longrightarrow 0}\left(\frac{1-\sin x\cos x}{1+\sin x\cos x}\right)^{\frac{1}{\sin x}}=[1^\infty]=e^\lambda$$
$$\lambda=\lim_{x\longrightarrow 0}\frac{1}{\sin x}\left(\frac{1-\sin x\cos x}{1+\sin x\cos x}-1\right)=\lim_{x\longrightarrow 0}\frac{1-\sin x\cos x-1-\sin x\cos x}{\sin x(1+\sin x\cos x)}=$$
$$\lim_{x\longrightarrow 0}\frac{-2\sin x\cos x}{\sin x(1+\sin x\cos x)}=\lim_{x\longrightarrow 0}\frac{-2\cos x}{1+\sin x\cos x}=-2$$

Luego $L=e^{-2}$.

b)

$$\lim_{x\longrightarrow 4}\frac{\tan\left(\left(\frac{\pi}{8}+1\right)\sqrt{x}-2\right)}{x^2-16+ax}=\lim_{x\longrightarrow 4}\frac{\tan\left(\left(\frac{\pi}{8}+1\right)2-2\right)}{4a}=\lim_{x\longrightarrow 4}\frac{\tan\left(\frac{\pi}{4}\right)}{4a}=$$

$$\frac{1}{4a}=\frac{1}{32}\Longrightarrow 4a=32\Longrightarrow a=8$$

Problema 10 Calcule los siguientes límites:

a) $\lim_{x\longrightarrow 0}\frac{\ln(3+x)-\ln(3-x)}{2x}$.

b) $\lim_{x\longrightarrow +\infty}(\sqrt{x+1}-\sqrt{x+2})$.

Solución:

a) $\lim_{x\longrightarrow 0}\frac{\ln(3+x)-\ln(3-x)}{2x}=\left[\frac{0}{0}\right]=\lim_{x\longrightarrow 0}\frac{\frac{1}{3+x}-\frac{-1}{3-x}}{2}=\frac{\frac{1}{3}+\frac{1}{3}}{2}=\frac{1}{3}$.

b) $\lim_{x\longrightarrow +\infty}(\sqrt{x+1}-\sqrt{x+2})=[\infty-\infty]=\lim_{x\longrightarrow +\infty}\frac{(\sqrt{x+1}-\sqrt{x+2})(\sqrt{x+1}+\sqrt{x+2})}{(\sqrt{x+1}+\sqrt{x+2})}=$
$\lim_{x\longrightarrow +\infty}\frac{(\sqrt{x+1})^2-(\sqrt{x+2})^2}{(\sqrt{x+1}+\sqrt{x+2})}=\lim_{x\longrightarrow +\infty}\frac{-1}{(\sqrt{x+1}+\sqrt{x+2})}=0$