Examen de Matemáticas 2ºBachillerato(CN)
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Problema 1 Sea la matriz

$$A = \begin{pmatrix} m & -m & 3 \\ 2 & 0 & 1 \\ m & 3 & -7 \end{pmatrix}$$

a) Calcular los valores de m para los que la matriz A es inversible.

b) Calcular A^{-1} para $m = 0$.

Solución:

a)

$$\begin{vmatrix} m & -m & 3 \\ 2 & 0 & 1 \\ m & 3 & -7 \end{vmatrix} = -m^2 - 17m + 18 = 0 \Longrightarrow m = -18, \quad m = 1$$

Si $m = -18$ o $m = 1 \Longrightarrow |A| = 0 \Longrightarrow \nexists A^{-1}$.

Si $m \neq -18$ y $m \neq 1 \Longrightarrow |A| \neq 0 \Longrightarrow \exists A^{-1}$.

b) $A = \begin{pmatrix} 0 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 3 & -7 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} -1/6 & 1/2 & 0 \\ 7/9 & 0 & 1/3 \\ 1/3 & 0 & 0 \end{pmatrix}$

Problema 2 Resolver la ecuación matricial $AX + BX = 2I + CX$. Donde

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}; \quad B = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

Solución:

$$AX + BX = 2I + CX \Longrightarrow X = 2(A + B - C)^{-1}$$

$$A + B - C = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -4 & 7 \end{pmatrix}$$

$$(A + B - C)^{-1} = \begin{pmatrix} -7/6 & -5/6 \\ -2/3 & -1/3 \end{pmatrix}$$

$$X = 2(I - B + C)^{-1} = \begin{pmatrix} -7/3 & -5/3 \\ -4/3 & -2/3 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\left|\begin{array}{cccc} x & 1 & 0 & 1 \\ 0 & x & 1 & 1 \\ 1 & 1 & x & 0 \\ 1 & 0 & 1 & x \end{array}\right|$$

Solución:

$$\left|\begin{array}{cccc} x & 1 & 0 & 1 \\ 0 & x & 1 & 1 \\ 1 & 1 & x & 0 \\ 1 & 0 & 1 & x \end{array}\right| = \left[\begin{array}{c} F_1+F_2+F_3+F_4 \\ F_2 \\ F_3 \\ F_4 \end{array}\right] = \left|\begin{array}{cccc} x+2 & x+2 & x+2 & x+2 \\ 0 & x & 1 & 1 \\ 1 & 1 & x & 0 \\ 1 & 0 & 1 & x \end{array}\right| =$$

$$(x+2)\left|\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 0 & x & 1 & 1 \\ 1 & 1 & x & 0 \\ 1 & 0 & 1 & x \end{array}\right| = \left[\begin{array}{c} C_1 \\ C_2-C_1 \\ C_3+C_1 \\ C_4-C_1 \end{array}\right] = (x+2)\left|\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & x & 1 & 1 \\ 1 & 0 & x-1 & -1 \\ 1 & -1 & 0 & x-1 \end{array}\right| =$$

$$(x+2)\left|\begin{array}{ccc} x & 1 & 1 \\ 0 & x-1 & -1 \\ -1 & 0 & x-1 \end{array}\right| = \left[\begin{array}{c} F_1+F_2 \\ F_2 \\ F_3 \end{array}\right] = (x+2)\left|\begin{array}{ccc} x & x & 0 \\ 0 & x-1 & -1 \\ -1 & 0 & x-1 \end{array}\right| =$$

$$x(x+2)\left|\begin{array}{ccc} 1 & 1 & 0 \\ 0 & x-1 & -1 \\ -1 & 0 & x-1 \end{array}\right| = \left[\begin{array}{c} F_1 \\ F_2 \\ F_3+F_1 \end{array}\right] = x(x+2)\left|\begin{array}{ccc} 1 & 1 & 0 \\ 0 & x-1 & -1 \\ 0 & 1 & x-1 \end{array}\right| =$$

$$x(x+2)\left|\begin{array}{cc} x-1 & -1 \\ 1 & x-1 \end{array}\right| = x(x-2)\left[(x-1)^2+1\right] =$$

$$x(x-2)(x^2-2x+2) = x^4-2x^2+4x$$