

Examen de Matemáticas 2º Bachillerato (CS)
Octubre 2014

Problema 1 Calcular el rango de la matriz

$$A = \begin{pmatrix} 2 & 1 & -1 & 3 \\ 3 & 1 & 0 & -1 \\ -1 & 1 & -4 & 15 \end{pmatrix}$$

Solución:

$$|A_1| = \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & 0 \\ -1 & 1 & -4 \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 1 & -1 \\ -1 & 1 & 15 \end{vmatrix} = 0$$
$$|A_3| = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 0 & -1 \\ -1 & -4 & 15 \end{vmatrix} = 0, \quad |A_4| = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & -4 & 15 \end{vmatrix} = 0$$

Como

$$\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -1 \neq 0 \implies \text{Rango}(A) = 2$$

Problema 2 Sea la matriz

$$A = \begin{pmatrix} m & -m & 2 \\ 2 & 2 & m \\ 1 & -5 & 5 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & -m & 2 \\ 2 & 2 & m \\ 1 & -5 & 5 \end{vmatrix} = 4m^2 + 20m - 24 = 0 \implies m = 1, \quad m = -6$$

Si $m = 1$ o $m = -6 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq -6 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 2 & 0 \\ 1 & -5 & 5 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -5/12 & 5/12 & 1/6 \\ 5/12 & 1/12 & -1/6 \\ 1/2 & 0 & 0 \end{pmatrix}$$

Problema 3 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{1000}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{1000} = \begin{pmatrix} 1 & 0 & 0 \\ 1000 & 1 & 1000 \\ 0 & 0 & 1 \end{pmatrix}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$$

Solución:

Llamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix} \implies$$

$$\begin{pmatrix} a - 2c & b - 2d \\ 3a & 3b \end{pmatrix} = \begin{pmatrix} a + 3b & -2a \\ c + 3d & -2c \end{pmatrix} \implies$$

$$\begin{cases} a - 2c = a + 3b \implies -2c = 3b \implies b = -2/3c = -2/3(3a - 3d) = -2a + 2d \\ b - 2d = -2a \implies -2/3c - 2d = -2a \implies -2c - 6d = -6a \implies 3a = c + 3d \\ 3a = c + 3d \implies c = 3a - 3d \\ 3b = -2c \implies -2c = 3b \implies b = -2/3c \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} a & -2a + 2d \\ 3a - 3d & d \end{pmatrix}.$$