

Examen de Matemáticas 2º Bachillerato (CN)
Marzo 2009

Calcular las siguientes integrales:

1. $\int_{-2}^{-1} \frac{dx}{(x^2 - x)(x - 1)}$ (Andalucía Junio 2008)

Solución:

$$\int_{-2}^{-1} \frac{dx}{(x^2 - x)(x - 1)} = \int_{-2}^{-1} \frac{dx}{x(x - 1)^2}$$

Hacemos la descomposición en fracciones simples:

$$\frac{1}{x(x - 1)^2} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} = \frac{A(x - 1)^2 + Bx(x - 1) + Cx}{x(x - 1)^2}$$

$$1 = A(x - 1)^2 + Bx(x - 1) + Cx$$

Si $x = 0 \implies A = 1$

Si $x = 1 \implies C = 1$

Si $x = 2 \implies A + 2B + 2C = 1 \implies C = -1$ luego:

$$\begin{aligned} \int_{-2}^{-1} \frac{dx}{(x^2 - x)(x - 1)} &= \int_{-2}^{-1} \left(\frac{1}{x} - \frac{1}{x - 1} + \frac{1}{(x - 1)^2} \right) dx = \\ &= \left[\ln|x| - \ln|x - 1| - \frac{1}{x - 1} \right]_{-2}^{-1} = \frac{1}{6} + \ln \frac{3}{4} \end{aligned}$$

2. $\int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx$ (Castilla-La Mancha Junio 2008)

Solución:

$$\int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx = \int \left(2x + 1 + \frac{2x}{(x - 2)(x - 3)} \right) dx$$

Hacemos la descomposición en fracciones simples:

$$\frac{2x}{(x - 2)(x - 3)} = \frac{A}{x - 2} + \frac{B}{x - 3} = \frac{A(x - 3) + B(x - 2)}{(x - 2)(x - 3)}$$

$$2x = A(x - 3) + B(x - 2)$$

Si $x = 2 \implies A = -4$

Si $x = 3 \implies B = 6$

$$\begin{aligned} \int \frac{2x^3 - 9x^2 + 9x + 6}{x^2 - 5x + 6} dx &= \int \left(2x + 1 - \frac{4}{x - 2} + \frac{6}{x - 3} \right) dx = \\ &= x^2 + x - 4 \ln|x - 2| + 6 \ln|x - 3| + C \end{aligned}$$

3. $\int_0^\pi e^x \sin x \, dx$ (Castilla-La Mancha Junio 2008)

Solución:

Se trata de una integral por partes, hacemos

$$\begin{aligned} u = \sin x &\implies du = \cos x \, dx \\ dv = e^x \, dx &\implies v = e^x \end{aligned}$$

$$A = \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

Volvemos a integrar por partes:

$$\begin{aligned} u = \cos x &\implies du = -\sin x \, dx \\ dv = e^x \, dx &\implies v = e^x \end{aligned}$$

$$\begin{aligned} A &= \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx = \\ &e^x \sin x - e^x \cos x - \int e^x \sin x \, dx \end{aligned}$$

Es decir:

$$A = e^x \sin x - e^x \cos x - A \implies A = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\int_0^\pi e^x \sin x \, dx = \left[\frac{1}{2} e^x (\sin x - \cos x) \right]_0^\pi = \frac{1}{2} (e^\pi + 1)$$

Calcular los siguientes límites:

1. $\lim_{x \rightarrow 0} \frac{x^3 - 8x^2 + 7x}{x^2 - x}$ (Castilla-La Mancha Junio 2008)

Solución: $\lim_{x \rightarrow 0} \frac{x(x^2 - 8x + 7)}{x(x - 1)} = -7$

2. $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{2x}{\pi} + \cos x \right)^{1/\cos x}$ Castilla-La Mancha (Junio 2008)

Solución:

$$\ln \lambda = \lim_{x \rightarrow \frac{\pi}{2}} \ln \left(\left(\frac{2x}{\pi} + \cos x \right)^{1/\cos x} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln \left(\frac{2x}{\pi} + \cos x \right)}{\cos x} =$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{2/\pi - \sin x}{2x/\pi + \cos x}}{-\sin x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{2}{\pi} - \sin x}{-\sin x \left(\frac{2x}{\pi} + \cos x \right)} = 1 - \frac{2}{\pi}$$

$$\ln \lambda = 1 - \frac{2}{\pi} \implies \lambda = e^{1 - \frac{2}{\pi}}$$