Examen de Matemáticas 1º de Bachillerato CN
Marzo 2025

Problema 1 Calcular los siguientes límites:

a) $\lim_{x\longrightarrow\infty}\left(\sqrt{7x^2+x-3}-\sqrt{7x^2-3x+5}\right)$

b) $\lim_{x\longrightarrow 6}\dfrac{\sqrt{x^2+5}-\sqrt{7x-1}}{x-6}$

c) $\lim_{x\longrightarrow 1}\dfrac{x^4+7x^3-11x^2+x+2}{2x^3+x^2-x-2}$

d) $\lim_{x\longrightarrow 0}\dfrac{5\cos x-xe^x-5}{x\cos x}$

e) $\lim_{x\longrightarrow\infty}\dfrac{2e^{4x}+x^2-1}{e^{4x}+3x+2}$

f) $\lim_{x\longrightarrow 0}\dfrac{3x\cos x+4xe^x}{\cos x-e^x}$

g) $\lim_{x\longrightarrow\infty}\dfrac{\ln(5x^2+3)}{\ln(x^2+1)}$

Solución:

a) $\lim_{x\longrightarrow\infty}\left(\sqrt{7x^2+x-3}-\sqrt{7x^2-3x+5}\right)=\dfrac{2\sqrt{7}}{7}$

b) $\lim_{x\longrightarrow 6}\dfrac{\sqrt{x^2+5}-\sqrt{7x-1}}{x-6}=\dfrac{5\sqrt{41}}{82}$

c) $\lim_{x\longrightarrow 1}\dfrac{x^4+7x^3-11x^2+x+2}{2x^3+x^2-x-2}=\dfrac{4}{7}$

d) $\lim_{x\longrightarrow 0}\dfrac{5\cos x-xe^x-5}{x\cos x}=-1$

e) $\lim_{x\longrightarrow\infty}\dfrac{2e^{4x}+x^2-1}{e^{4x}+3x+2}=2$

f) $\lim_{x\longrightarrow 0}\dfrac{3x\cos x+4xe^x}{\cos x-e^x}=-7$

g) $\lim_{x\longrightarrow\infty}\dfrac{\ln(5x^2+3)}{\ln(x^2+1)}=1$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y = \ln \sqrt[7]{\dfrac{x^3 \sin^3(2x)}{e^{5x} \cos x^2}}$

b) $y = (x^2+1)^{\sin(2x)}$

c) $y = (\arccos x)^{8x-2}$

d) $y = \log_3 \dfrac{3x^2-3}{\sqrt{x^2+3}}$

e) $y = \sqrt[5]{\dfrac{x^2-5}{\cos^2(3x)}}$

f) $y = \sec^2(x^3-2)\log_3(x^2+1)$

g) $y = 5^{\arctan(x^2-1)}\tan^2(x+5)$

Solución:

a) $y = \ln \sqrt[7]{\dfrac{x^3 \sin^3(2x)}{e^{5x} \cos x^2}} = \dfrac{1}{7}\left(3\ln x + 3\ln\sin(2x) - (5x)\ln e - \ln(\cos x^2)\right) \Longrightarrow$

$$y' = \frac{1}{7}\left(\frac{3}{x} + 3\frac{2\cos(2x)}{\sin(2x)} - 5 - \frac{-2x\sin x^2}{\cos x^2}\right)$$

b) $y = (x^2+1)^{\sin(2x)} \Longrightarrow y' = (x^2+1)^{\sin(2x)}\left(2\cos(2x)\ln(x^2+1) + \sin(2x)\dfrac{2x}{x^2+1}\right)$

c) $y = (\arccos x)^{8x-2} \Longrightarrow y' = (\arccos x)^{8x-2}\left(8\ln(\arccos x) + (8x-2)\dfrac{\frac{-1}{\sqrt{1-x^2}}}{\arccos x}\right)$

d) $y = \log_3 \dfrac{3x^2-3}{\sqrt{x^2+3}} = \log_3(3x^2-3) - \dfrac{1}{2}\log_3(x^2+3) \Longrightarrow y' = \dfrac{6x}{(3x^2-3)\ln 3} - \dfrac{1}{2}\dfrac{2x}{(x^2+3)\ln 3}$

e) $y = \sqrt[5]{\dfrac{x^2-5}{\cos^2(3x)}} \Longrightarrow y' = \dfrac{1}{5}\left(\dfrac{x^2-5}{\cos^2(3x)}\right)^{-4/5}\left(\dfrac{2x\cos^2(3x) - (x^2-5)(2\cos(3x)(-3\sin(3x)))}{\cos^4(3x)}\right)$

f) $y = \sec^2(x^3-2)\log_3(x^2+1) \Longrightarrow y' = 3x^2\sec^2(x^3-2)\tan(x^3-2)\log_3(x^2+1) + \sec^2(x^3-2)\dfrac{2x}{(x^2+1)\ln 3}$

g) $y = 5^{\arctan(x^2-1)}\tan^2(x+5) \Longrightarrow y' = \dfrac{2x}{1+(x^2-1)^2}5^{\arctan(x^2-1)}\ln 5\tan^2(x+5) + 5^{\arctan(x^2-1)}2\tan(x+5)\dfrac{1}{\cos^2(x+5)}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \dfrac{2x^2 - 3}{2x + 1}$ en el punto $x = 0$.

b) $f(x) = (x^2 - 1)e^{3x}$ en el punto $x = 0$.

Solución:

a) $b = f(a) \Longrightarrow b = f(0) = -3$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(2x^2 + 2x + 3)}{(2x + 1)^2} \Longrightarrow m = f'(0) = 6$$

Recta Tangente: $y + 3 = 6x$

Recta Normal: $y + 3 = -\dfrac{1}{6}x$

b) $b = f(a) \Longrightarrow b = f(0) = -1$ e $y - b = m(x - a)$

$$f'(x) = (3x^2 + 2x - 3)e^{3x} \Longrightarrow m = f'(0) = -3$$

Recta Tangente: $y + 1 = -3x$

Recta Normal: $y + 1 = \dfrac{1}{3}x$

Problema 4 Calcular las siguientes integrales:

a) $\displaystyle\int 2xe^{3x^2+7}\,dx$

b) $\displaystyle\int \frac{5x}{x^2 + 7}\,dx$

c) $\displaystyle\int 5x^3\cos(x^4 + 1)\,dx$

d) $\displaystyle\int \frac{x}{1 + x^4}\,dx$

e) $\displaystyle\int \frac{3x^2 - 2x^2\cos x - 6x^2e^x + 5x}{x^2}\,dx$

f) $\displaystyle\int \frac{2x^5 - 4x^4 - 3\sqrt[5]{x^3} - 7x}{x^2}\,dx$

Solución:

a) $\displaystyle\int 2xe^{3x^2+7}\,dx = \frac{1}{3}e^{3x^2+7} + C$

b) $\displaystyle\int \frac{5x}{x^2+7}\,dx = \frac{5}{2}\ln|x^2+7| + C$

c) $\displaystyle\int 5x^3\cos(x^4+1)\,dx = \frac{5}{4}\sin(x^4+1) + C$

d) $\displaystyle\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\arctan x^2 + C$

e) $\displaystyle\int \frac{3x^2-2x^2\cos x-6x^2e^x+5x}{x^2}\,dx = 3x+2\sin x-6e^x+5\ln|x| + C$

f) $\displaystyle\int \frac{2x^5-4x^4-3\sqrt[5]{x^3}-7x}{x^2}\,dx = \frac{x^4}{2}-\frac{4x^3}{3}+\frac{15x^{-2/5}}{2}-7\ln|x| + C$