Examen de Matemáticas 1º de Bachillerato CN
Marzo 2025

Problema 1 Calcular las siguientes integrales:

a) $\displaystyle\int \frac{x^2}{1-2x^3}\,dx$

b) $\displaystyle\int 9x(5x^2-1)^{20}\,dx$

c) $\displaystyle\int \frac{3x^2\cos x - x^2e^x + 3x - 1}{x^2}\,dx$

d) $\displaystyle\int \frac{5x^3 - 2\sqrt[5]{x^3} - 2x}{x^2}\,dx$

e) $\displaystyle\int \frac{x^3\sin(x^2-5) + x^3e^{3x^2+1} - 5x + 1}{x^2}\,dx$

f) $\displaystyle\int \frac{-11}{1+x^2}\,dx$

Solución:

a) $\displaystyle\int \frac{x^2}{1-2x^3}\,dx = -\frac{1}{6}\ln|1-2x^3| + C$

b) $\displaystyle\int 9x(5x^2-1)^{20}\,dx = \frac{3(5x^2-1)^{21}}{70} + C$

c) $\displaystyle\int \frac{3x^2\cos x - x^2e^x + 3x - 1}{x^2}\,dx = 3\sin x - e^x + 3\ln|x| + \frac{1}{x} + C$

d) $\displaystyle\int \frac{5x^3 - 2\sqrt[5]{x^3} - 2x}{x^2}\,dx = \frac{5x^2}{2} + \frac{5}{x^{2/5}} - 2\ln|x| + C$

e) $\displaystyle\int \frac{x^3\sin(x^2-5) + x^3e^{3x^2+1} - 5x + 1}{x^2}\,dx = -\frac{\cos(x^2-5)}{2} - \frac{e^{3x^2+1}}{6} - \frac{1}{x} - 5\ln|x| + C$

f) $\displaystyle\int \frac{-11}{1+x^2}\,dx = -11\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y = \ln\sqrt[7]{\dfrac{x^4\cos(x^2-1)}{e^{x^2+5}\sin x}}$

b) $y = (\sin x)^{x^2-5}$

c) $y = \dfrac{\arctan(x^2-6)(3x+1)}{x^2-1}$

d) $y = \csc(5x+2)^2 \sec^2(x^2+1)$

e) $y = 4^{\cos^2 x - \sin x} \log_5(5x^2 + \cos x)$

f) $y = (\sqrt{x^2-2})^{\arctan x}$

Solución:

a) $y = \ln \sqrt[7]{\dfrac{x^4 \cos(x^2-1)}{e^{x^2+5} \sin x}} = \dfrac{1}{7}\left(4 \ln x + \ln \cos(x^2-1) - (x^2+5) \ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{7}\left(\frac{4}{x} + \frac{-2x \sin(x^2-1)}{\cos(x^2-1)} - 2x - \frac{\cos x}{\sin x}\right)$$

b) $y = (\sin x)^{x^2-5} \Longrightarrow y' = (\sin x)^{x^2-5}\left(2x \ln \sin x + (x^2-5)\dfrac{\cos x}{\sin x}\right)$

c) $y = \dfrac{\arctan(x^2-6)(3x+1)}{x^2-1} \Longrightarrow$

$y' = \dfrac{\left(\frac{2x}{1+(x^2-6)^2}(3x+1) + 3\arctan(x^2-6)\right)(x^2-1) - \arctan(x^4-6)(3x+1)2x}{(x^2-1)^2}$

d) $y = \csc(5x+2)^2 \sec^2(x^2+1) \Longrightarrow y' = -10(5x+2) \csc(5x+2)^2 \cot(5x+2)^2 \sec^2(x^2+1) + \csc(5x+2)^2 2 \sec(x^2+1) 2x \sec(x^2+1) \tan(x^2+1)$

e) $y = 4^{\cos^2 x - \sin x} \log_5(5x^2+\cos x) \Longrightarrow y' = (2 \cos x(-\sin x) - \cos x) 4^{\cos^2 x - \sin x} \log_5(5x^2 + \cos x) + 4^{\cos^2 x - \sin x} \dfrac{10x - \sin x}{(5x^2 + \cos x) \ln 5}$

f) $y = (\sqrt{x^2-2})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2-2})^{\arctan x}\left(\dfrac{1}{1+x^2} \ln \sqrt{x^2-2} + \arctan x \dfrac{\frac{2x}{2\sqrt{x^2-2}}}{\sqrt{x^2-2}}\right)$

Problema 3 Calcular los siguientes límites:

a) $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{3x^2+6x+2} - \sqrt{3x^2-1}\right)$

b) $\lim\limits_{x \longrightarrow 1} \dfrac{3x^4 - 11x^3 - 9x^2 + 59x - 42}{3x^3 + 10x^2 - 23x + 10}$

c) $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{x^2+2} - \sqrt{4x-1}}{x-3}$

d) $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{4x^2+x-2}{4x^2-1}\right)^{7x}$

e) $\lim_{x \longrightarrow \infty} \dfrac{e^{3x^2-x+7}}{5x+1}$

f) $\lim_{x \longrightarrow \infty} \dfrac{e^{2x-1}-2}{e^{2x+1}-1}$

g) $\lim_{x \longrightarrow 0} \dfrac{\sin^2 x + 3x}{5x \cos x}$

Solución:

a) $\lim_{x \longrightarrow \infty} \left(\sqrt{3x^2+6x+2} - \sqrt{5x^2-1} \right) = \sqrt{3}$

b) $\lim_{x \longrightarrow 1} \dfrac{3x^4 - 11x^3 - 9x^2 + 59x - 42}{3x^3 + 10x^2 - 23x + 10} = \dfrac{10}{3}$

c) $\lim_{x \longrightarrow 3} \dfrac{\sqrt{x^2+2} - \sqrt{4x-1}}{x-3} = \dfrac{2\sqrt{11}}{11}$

d) $\lim_{x \longrightarrow \infty} \left(\dfrac{4x^2+x-2}{4x^2-1} \right)^{7x} = e^{7/4}$

e) $\lim_{x \longrightarrow \infty} \dfrac{e^{3x^2-x+7}}{5x+1} = \infty$

f) $\lim_{x \longrightarrow \infty} \dfrac{e^{2x-1}-2}{e^{2x+1}-1} = e^{-2}$

g) $\lim_{x \longrightarrow 0} \dfrac{\sin^2 x + 3x}{5x \cos x} = \dfrac{3}{5}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \dfrac{4x-7}{x+1}$ en el punto $x = 2$.

b) $f(x) = (x+7)e^{x-1}$ en el punto $x = 1$.

Solución:

a) $b = f(a) \Longrightarrow b = f(2) = \dfrac{1}{3}$ e $y - b = m(x-a)$

$$f'(x) = \frac{11}{(x+1)^2} \Longrightarrow m = f'(2) = \frac{11}{9}$$

Recta Tangente: $y - \dfrac{1}{3} = \dfrac{11}{9}(x-2)$

Recta Normal: $y - \dfrac{1}{3} = -\dfrac{9}{11}(x-2)$

b) $b = f(a) \Longrightarrow b = f(1) = 8$ e $y - b = m(x - a)$

$$f'(x) = (x + 8)e^{x-1} \Longrightarrow m = f'(1) = 9$$

Recta Tangente: $y - 8 = 9(x - 1)$

Recta Normal: $y - 8 = -\dfrac{1}{9}(x - 1)$