Examen de Matemáticas 1º de Bachillerato CN
Marzo 2024

Problema 1 Calcular los siguientes límites:

a) $\lim_{x\longrightarrow\infty}\left(\sqrt{3x^2+5x-3}-\sqrt{3x^2+x-1}\right)$

b) $\lim_{x\longrightarrow 6}\dfrac{\sqrt{x^2-5}-\sqrt{5x+1}}{x-6}$

c) $\lim_{x\longrightarrow 1}\dfrac{5x^4-24x^3+30x^2-8x-3}{2x^4-13x^3+15x^2+x-5}$

d) $\lim_{x\longrightarrow 0}\dfrac{2\cos x-xe^2x-2}{3x\cos x}$

e) $\lim_{x\longrightarrow\infty}\dfrac{5e^{3x}+x^2+1}{e^{3x}+5x-1}$

f) $\lim_{x\longrightarrow 0}\dfrac{5x\cos x-3xe^x}{\cos x-e^x}$

g) $\lim_{x\longrightarrow\infty}\dfrac{\ln(7x^2+5)}{\ln(x^2-2)}$

Solución:

a) $\lim_{x\longrightarrow\infty}\left(\sqrt{3x^2+5x-3}-\sqrt{3x^2+x-1}\right)=\dfrac{2\sqrt{3}}{3}$

b) $\lim_{x\longrightarrow 6}\dfrac{\sqrt{x^2-5}-\sqrt{5x+1}}{x-6}=\dfrac{7\sqrt{31}}{62}$

c) $\lim_{x\longrightarrow 1}\dfrac{5x^4-24x^3+30x^2-8x-3}{2x^4-13x^3+15x^2+x-5}=1$

d) $\lim_{x\longrightarrow 0}\dfrac{2\cos x-xe^2x-2}{3x\cos x}=-\dfrac{1}{3}$

e) $\lim_{x\longrightarrow\infty}\dfrac{5e^{3x}+x^2+1}{e^{3x}+5x-1}=5$

f) $\lim_{x\longrightarrow 0}\dfrac{5x\cos x-3xe^x}{\cos x-e^x}=-2$

g) $\lim_{x\longrightarrow\infty}\dfrac{\ln(7x^2+5)}{\ln(x^2-2)}=1$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y = \ln \sqrt[5]{\dfrac{x^2 \sin^3(2x)}{e^{2x} \cos x^2}}$

b) $y = (x^2+5)^{\sin(3x)}$

c) $y = (\arcsin x)^{2x-3}$

d) $y = \log_7 \dfrac{5x^2+2}{\sqrt{x^2-7}}$

e) $y = \sqrt[7]{\dfrac{x^2+2}{\cos^2(2x)}}$

f) $y = \sec^2(x^3+1)\log_5(x^2-9)$

g) $y = 5^{\arctan(x^2+6)}\tan^2(x-1)$

Solución:

a) $y = \ln \sqrt[5]{\dfrac{x^2 \sin^3(2x)}{e^{2x} \cos x^2}} = \dfrac{1}{5}\left(2\ln x + 3\ln\sin(2x) - (2x)\ln e - \ln(\cos x^2)\right) \Longrightarrow$

$$y' = \frac{1}{5}\left(\frac{2}{x} + 3\frac{2\cos(2x)}{\sin(2x)} - 2 - \frac{-2x\sin x^2}{\cos x^2}\right)$$

b) $y = (x^2+5)^{\sin(3x)} \Longrightarrow y' = (x^2+5)^{\sin(3x)}\left(3\cos(3x)\ln(x^2+5) + \sin(3x)\dfrac{2x}{x^2+5}\right)$

c) $y = (\arcsin x)^{2x-3} \Longrightarrow y' = (\arcsin x)^{2x-3}\left(2\ln(\arcsin x) + (2x-3)\dfrac{\frac{1}{\sqrt{1-x^2}}}{\arcsin x}\right)$

d) $y = \log_7 \dfrac{5x^2+2}{\sqrt{x^2-7}} = \log_7(5x^2+2) - \dfrac{1}{2}\log_7(x^2-7) \Longrightarrow y' = \dfrac{10x}{(5x^2+2)\ln 7} - \dfrac{1}{2}\dfrac{2x}{(x^2-7)\ln 7}$

e) $y = \sqrt[7]{\dfrac{x^2+2}{\cos^2(2x)}} \Longrightarrow y' = \dfrac{1}{7}\left(\dfrac{x^2+2}{\cos^2(2x)}\right)^{-6/7}\left(\dfrac{2x\cos^2(2x) - (x^2+2)(2\cos(2x)(-2\sin(2x)))}{\cos^4(2x)}\right)$

f) $y = \sec^2(x^3+1)\log_5(x^2-9) \Longrightarrow y' = 3x^2\sec^2(x^3+1)\tan(x^3+1)\log_5(x^2-9) + \sec^2(x^3+1)\dfrac{2x}{(x^2-9)\ln 5}$

g) $y = 5^{\arctan(x^2+6)}\tan^2(x-1) \Longrightarrow y' = \dfrac{2x}{1+(x^2+6)^2}5^{\arctan(x^2+6)}\ln 5\tan^2(x-1) + 5^{\arctan(x^2+6)}2\tan(x-1)\dfrac{1}{\cos^2(x-1)}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \dfrac{5x^2-4}{2x-1}$ en el punto $x = 0$.

b) $f(x) = (x^2+3)e^{3x}$ en el punto $x = 0$.

Solución:

a) $b = f(a) \Longrightarrow b = f(0) = 4$ e $y - b = m(x-a)$

$$f'(x) = \frac{2(5x^2-5x+4)}{(2x-1)^2} \Longrightarrow m = f'(0) = 8$$

Recta Tangente: $y - 4 = 8x$

Recta Normal: $y - 4 = -\dfrac{1}{8}x$

b) $b = f(a) \Longrightarrow b = f(0) = 3$ e $y - b = m(x-a)$

$$f'(x) = (3x^2+2x+9)e^{3x} \Longrightarrow m = f'(0) = 9$$

Recta Tangente: $y - 3 = 9x$

Recta Normal: $y - 3 = -\dfrac{1}{9}x$

Problema 4 Calcular las siguientes integrales:

a) $\displaystyle\int 5xe^{4x^2-8}\,dx$

b) $\displaystyle\int \frac{7x}{x^2-8}\,dx$

c) $\displaystyle\int 7x^3\cos(3x^4-1)\,dx$

d) $\displaystyle\int \frac{3x}{1+x^4}\,dx$

e) $\displaystyle\int \frac{5x^2-6x^2\cos x-8x^2e^x-5x}{x^2}\,dx$

f) $\displaystyle\int \frac{3x^5+6x^4-3\sqrt[5]{x^3}+8x}{x^2}\,dx$

Solución:

a) $\displaystyle\int 5xe^{4x^2-8}\,dx = \frac{5}{8}e^{4x^2-8} + C$

b) $\displaystyle\int \frac{7x}{x^2-8}\,dx = \frac{7}{2}\ln|x^2-8| + C$

c) $\displaystyle\int 7x^3\cos(3x^4-1)\,dx = \frac{7}{12}\sin(3x^4-1) + C$

d) $\displaystyle\int \frac{3x}{1+x^4}\,dx = \frac{3}{2}\arctan x^2 + C$

e) $\displaystyle\int \frac{5x^2-6x^2\cos x-8x^2e^x-5x}{x^2}\,dx = 5x - 6\sin x - 8e^x - 5\ln|x| + C$

f) $\displaystyle\int \frac{3x^5+6x^4-3\sqrt[5]{x^3}+8x}{x^2}\,dx = \frac{3x^4}{4} + 2x^3 + \frac{15x^{-2/5}}{2} + 8\ln|x| + C$