Examen de Matemáticas 1º de Bachillerato CN
Marzo 2024

Problema 1 Calcular las siguientes integrales:

a) $\displaystyle\int \frac{5x^2}{1-4x^3}\,dx$

b) $\displaystyle\int 5x(3x^2+8)^{23}\,dx$

c) $\displaystyle\int \frac{3x^2\cos x+5x^2e^x-3x-2}{x^2}\,dx$

d) $\displaystyle\int \frac{5x^3-2\sqrt[5]{x^2}-x}{x^2}\,dx$

e) $\displaystyle\int \frac{5x^3\sin(x^2+2)+x^3e^{2x^2-1}+7x+1}{x^2}\,dx$

f) $\displaystyle\int \frac{-8}{1+x^2}\,dx$

Solución:

a) $\displaystyle\int \frac{5x^2}{1-4x^3}\,dx = -\frac{5}{12}\ln|1-4x^3|+C$

b) $\displaystyle\int 5x(3x^2+8)^{23}\,dx = \frac{5(3x^2+8)^{24}}{144}+C$

c) $\displaystyle\int \frac{3x^2\cos x+5x^2e^x-3x-2}{x^2}\,dx = 3\sin x+5e^x-3\ln|x|+\frac{2}{x}+C$

d) $\displaystyle\int \frac{5x^3-2\sqrt[5]{x^2}-x}{x^2}\,dx = \frac{5x^2}{2}+\frac{10}{3x^{3/5}}-\ln|x|+C$

e) $\displaystyle\int \frac{5x^3\sin(x^2+2)+x^3e^{2x^2-1}+7x+1}{x^2}\,dx = -\frac{5\cos(x^2+2)}{2}+\frac{e^{2x^2-1}}{4}-\frac{1}{x}+7\ln|x|+ C$

f) $\displaystyle\int \frac{-8}{1+x^2}\,dx = -8\arctan x+C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y=\ln\sqrt[5]{\dfrac{x^2\cos(x^2-3)}{e^{x^2+1}\sin x}}$

b) $y = (\cos x)^{x^3+1}$

c) $y = \dfrac{\arctan(x^3+1)(5x+2)}{x^2+1}$

d) $y = \csc(3x-1)^2 \sec^2(x^2+2)$

e) $y = 9^{\sin^2 x - \cos x} \log_5(5x^2 + \cos x)$

f) $y = (\sqrt{x^2-1})^{\arctan x}$

Solución:

a) $y = \ln \sqrt[5]{\dfrac{x^2\cos(x^2-3)}{e^{x^2+1}\sin x}} = \dfrac{1}{5}\left(2\ln x + \ln\cos(x^2-3) - (x^2+1)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{5}\left(\frac{2}{x} + \frac{-2x\sin(x^2-3)}{\cos(x^2-3)} - 2x - \frac{\cos x}{\sin x}\right)$$

b) $y = (\cos x)^{x^3+1} \Longrightarrow y' = (\cos x)^{x^3+1}\left(3x^2\ln\cos x + (x^3+1)\dfrac{-\sin x}{\cos x}\right)$

c) $y = \dfrac{\arctan(x^3+1)(5x+2)}{x^2+1} \Longrightarrow$
$y' = \dfrac{\left(\frac{3x^2}{1+(x^3+1)^2}(5x+2) + 5\arctan(x^3+1)\right)(x^2+1) - \arctan(x^3+1)(5x+2)2x}{(x^2+1)^2}$

d) $y = \csc(3x-1)^2\sec^2(x^2+2) \Longrightarrow y' = -6(3x-1)\csc(3x-1)^2\cot(3x-1)^2\sec^2(x^2+2) + \csc(3x-1)^2 2\sec(x^2+2)2x\sec(x^2+2)\tan(x^2+2)$

e) $y = 9^{\sin^2 x-\cos x}\log_5(5x^2+\cos x) \Longrightarrow y' = (2\sin x\cos x + \sin x)9^{\sin^2 x - \cos x}\ln 9\log_5(5x^2+\cos x) + 9^{\sin^2 x-\cos x}\dfrac{10x-\sin x}{(5x^2+\cos x)\ln 5}$

f) $y = (\sqrt{x^2-1})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2-1})^{\arctan x}\left(\dfrac{1}{1+x^2}\ln\sqrt{x^2-1} + \arctan x\dfrac{\frac{2x}{2\sqrt{x^2-1}}}{\sqrt{x^2-1}}\right)$

Problema 3 Calcular los siguientes límites:

a) $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{2x^2+8x-1} - \sqrt{2x^2+5}\right)$

b) $\lim\limits_{x\longrightarrow 1}\dfrac{x^4+2x^3-28x^2+46x-21}{x^4-x^3-21x^2+41x-20}$

c) $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{x^2+4}-\sqrt{2x+7}}{x-3}$

d) $\lim_{x\longrightarrow\infty}\left(\frac{5x^2-2x+3}{5x^2-1}\right)^{2x}$

e) $\lim_{x\longrightarrow\infty}\frac{e^{3x^2-x+1}}{4x+1}$

f) $\lim_{x\longrightarrow\infty}\frac{e^{2x-7}-3}{e^{2x+5}-2}$

g) $\lim_{x\longrightarrow 0}\frac{\sin^2 x-7x}{3x\cos x}$

Solución:

a) $\lim_{x\longrightarrow\infty}\left(\sqrt{2x^2+8x-1}-\sqrt{2x^2+5}\right)=2\sqrt{2}$

b) $\lim_{x\longrightarrow 1}\frac{x^4+2x^3-28x^2+46x-21}{x^4-x^3-21x^2+41x-20}=\frac{8}{9}$

c) $\lim_{x\longrightarrow 3}\frac{\sqrt{x^2+4}-\sqrt{2x+7}}{x-3}=\frac{2\sqrt{13}}{13}$

d) $\lim_{x\longrightarrow\infty}\left(\frac{5x^2-2x+3}{5x^2-1}\right)^{2x}=e^{-4/5}$

e) $\lim_{x\longrightarrow\infty}\frac{e^{3x^2-x+1}}{4x+1}=\infty$

f) $\lim_{x\longrightarrow\infty}\frac{e^{2x-7}-3}{e^{2x+5}-2}=e^{-12}$

g) $\lim_{x\longrightarrow 0}\frac{\sin^2 x-7x}{3x\cos x}=-\frac{7}{3}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x)=\frac{6x-1}{x-1}$ en el punto $x=2$.

b) $f(x)=(x+1)e^{x-2}$ en el punto $x=2$.

Solución:

a) $b=f(a)\Longrightarrow b=f(2)=11$ e $y-b=m(x-a)$

$$f'(x)=-\frac{5}{(x-1)^2}\Longrightarrow m=f'(2)=-5$$

Recta Tangente: $y-11=-5(x-2)$

Recta Normal: $y-11=\frac{1}{5}(x-2)$

b) $b = f(a) \Longrightarrow b = f(2) = 3$ e $y - b = m(x - a)$

$$f'(x) = (x+2)e^{x-1} \Longrightarrow m = f'(2) = 4$$

Recta Tangente: $y - 3 = 4(x - 2)$

Recta Normal: $y - 3 = -\dfrac{1}{4}(x - 2)$