Examen de Matemáticas 1º de Bachillerato CN
Marzo 2022

Problema 1 Calcular las siguientes integrales:

a) $\displaystyle\int \frac{7x^2}{1+2x^3}\,dx$

b) $\displaystyle\int 3x(5x^2-1)^{18}\,dx$

c) $\displaystyle\int \frac{3x^2\cos x + 3x^2e^x - 5x + 2}{x^2}\,dx$

d) $\displaystyle\int \frac{2x^3 - 2\sqrt[5]{x^2} - x}{x^2}\,dx$

e) $\displaystyle\int \frac{5x^3\sin(3x^2-1) - 2x^3e^{7x^2-2} - 8x + 2}{x^2}\,dx$

f) $\displaystyle\int \frac{-8}{1+x^2}\,dx$

Solución:

a) $\displaystyle\int \frac{7x^2}{1+2x^3}\,dx = \frac{7}{6}\ln|1+2x^3| + C$

b) $\displaystyle\int 3x(5x^2-1)^{18}\,dx = \frac{3(5x^2-1)^{19}}{190} + C$

c) $\displaystyle\int \frac{3x^2\cos x + 3x^2e^x - 5x + 2}{x^2}\,dx = 3\sin x + 3e^x - 5\ln|x| - \frac{2}{x} + C$

d) $\displaystyle\int \frac{2x^3 - 2\sqrt[5]{x^2} - x}{x^2}\,dx = x^2 + \frac{10}{3x^{3/5}} - \ln|x| + C$

e) $\displaystyle\int \frac{5x^3\sin(3x^2-1) - 2x^3e^{7x^2-2} - 8x + 2}{x^2}\,dx = -\frac{5\cos(3x^2-1)}{6} - \frac{e^{7x^2-2}}{7} - \frac{2}{x} - 8\ln|x| + C$

f) $\displaystyle\int \frac{-8}{1+x^2}\,dx = -8\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y = \ln\sqrt[5]{\dfrac{x^4\cos(x^2-1)}{e^{x^2+3}\sin x}}$

b) $y = (\sin x)^{x^4-1}$

c) $y = \dfrac{\arctan(x^4+2)(3x-5)}{x^2+1}$

d) $y = \csc(3x-1)^2 \sec^2(x^2+2)$

e) $y = 5^{\cos^2 x - \sin x} \log_7(3x^2 + \cos x)$

f) $y = (\sqrt{x^2-5})^{\arctan x}$

Solución:

a) $y = \ln \sqrt[5]{\dfrac{x^4\cos(x^2-1)}{e^{x^2+3}\sin x}} = \dfrac{1}{5}\left(4\ln x + \ln\cos(x^2-1) - (x^2+3)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{3}\left(\frac{4}{x} + \frac{-2x\sin(x^2-1)}{\cos(x^2-1)} - 2x - \frac{\cos x}{\sin x}\right)$$

b) $y = (\sin x)^{x^4-1} \Longrightarrow y' = (\sin x)^{x^4-1}\left(4x^3\ln\sin x + (x^4-1)\dfrac{\cos x}{\sin x}\right)$

c) $y = \dfrac{\arctan(x^4+2)(3x-5)}{x^2+1} \Longrightarrow$
$y' = \dfrac{\left(\frac{4x^3}{1+(x^4+2)^2}(3x-5) + 3\arctan(x^4+2)\right)(x^2+1) - \arctan(x^4+2)(3x-5)2x}{(x^2+1)^2}$

d) $y = \csc(3x-1)^2\sec^2(x^2+2) \Longrightarrow y' = -6(3x-1)\csc(3x-1)^2\cot(3x-1)^2\sec^2(x^2+2) + \csc(3x-1)^2 2\sec(x^2+2)2x\sec(x^2+2)\tan(x^2+2)$

e) $y = 5^{\cos^2 x - \sin x}\log_7(3x^2+\cos x) \Longrightarrow y' = (2\cos x(-\sin x) - \cos x)5^{\cos^2 x - \sin x}\log_7(3x^2 + \cos x) + 5^{\cos^2 x - \sin x}\dfrac{6x - \sin x}{(3x^2+\cos x)\ln 7}$

f) $y = (\sqrt{x^2-5})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2-5})^{\arctan x}\left(\dfrac{1}{1+x^2}\ln\sqrt{x^2-5} + \arctan x\dfrac{\frac{2x}{2\sqrt{x^2-5}}}{\sqrt{x^2-5}}\right)$

Problema 3 Calcular los siguientes límites:

a) $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{3x^2-2x+3} - \sqrt{3x^2+8}\right)$

b) $\lim\limits_{x\longrightarrow 1}\dfrac{5x^4-7x^3+5x^2+2x-5}{x^3-3x^2+4x-2}$

c) $\lim\limits_{x\longrightarrow 3}\dfrac{\sqrt{x^2+4}-\sqrt{5x-2}}{x-3}$

d) $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{7x^2-x+2}{7x^2+5}\right)^{4x}$

e) $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{5x^2+7}}{2x+1}$

f) $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{2x+8}-3}{e^{5x+1}-1}$

g) $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x - 3x}{5x \cos x}$

Solución:

a) $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{3x^2-2x+3}-\sqrt{3x^2+8}\right) = -\dfrac{\sqrt{3}}{3}$

b) $\lim\limits_{x \longrightarrow 1} \dfrac{5x^4-7x^3+5x^2+2x-5}{x^3-3x^2+4x-2} = 11$

c) $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{x^2+4}-\sqrt{5x-2}}{x-3} = \dfrac{\sqrt{13}}{26}$

d) $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{7x^2-x+2}{7x^2+5}\right)^{4x} = e^{-4/7}$

e) $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{5x^2+7}}{2x+1} = \infty$

f) $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{2x+8}-3}{e^{5x+1}-1} = 0$

g) $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x - 3x}{5x \cos x} = -\dfrac{3}{5}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \dfrac{2x+1}{x-1}$ en el punto $x = 2$.

b) $f(x) = (x+5)e^{x-1}$ en el punto $x = 1$.

Solución:

a) $b = f(a) \Longrightarrow b = f(2) = 5$ e $y - b = m(x-a)$

$$f'(x) = -\frac{3}{(x-1)^2} \Longrightarrow m = f'(2) = -3$$

Recta Tangente: $y - 5 = -3(x-2)$

Recta Normal: $y - 5 = \dfrac{1}{3}(x-2)$

b) $b = f(a) \Longrightarrow b = f(1) = 6$ e $y - b = m(x - a)$

$$f'(x) = (x + 6)e^{x-1} \Longrightarrow m = f'(1) = 7$$

Recta Tangente: $y - 6 = 7(x - 1)$

Recta Normal: $y - 6 = -\dfrac{1}{7}(x - 1)$