# Examen de Matemáticas 1º de Bachillerato CS

**Marzo 2021**

**Problema 1** Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{7x^3-3x^2-3x+5}{2x^3-5x^2-2}$

2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{8x^2-3x+2}{4x^2-x+5}\right)^{x^3-7x-4}$

3. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{4x^2-3x+8}{4x^2+7}\right)^{5x-3}$

4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^3+3x-1}}{5x^2-4x+2}$

5. $\lim\limits_{x\longrightarrow 1} \dfrac{x^4-x^3-5x^2+8x-3}{x^4-6x^3+7x^2-10x+8}$

6. $\lim\limits_{x\longrightarrow 2} \dfrac{2x^3+x^2-13x+6}{3x^3-x^2-11x+2}$

7. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{2x^2+1}-\sqrt{9x+6}}{x-5}$

8. $\lim\limits_{x\longrightarrow 8} \dfrac{\sqrt{x^2+5}-\sqrt{9x-3}}{x-8}$

**Solución:**

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{7x^3-3x^2-3x+5}{2x^3-5x^2-2} = \dfrac{7}{2}$

2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{8x^2-3x+2}{4x^2-x+5}\right)^{x^3-7x-4} = \infty$

3. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{4x^2-3x+8}{4x^2+7}\right)^{5x-3} = e^{-15/4}$

4. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{4x^3+3x-1}}{5x^2-4x+2} = 0$

5. $\lim\limits_{x\longrightarrow 1} \dfrac{x^4-x^3-5x^2+8x-3}{x^4-6x^3+7x^2-10x+8} = \dfrac{1}{10}$

6. $\lim\limits_{x\longrightarrow 2} \dfrac{2x^3+x^2-13x+6}{3x^3-x^2-11x+2} = \dfrac{5}{7}$

7. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{2x^2+1}-\sqrt{9x+6}}{x-5} = \dfrac{11\sqrt{51}}{102}$

8. $\lim\limits_{x\longrightarrow 8} \dfrac{\sqrt{x^2+5}-\sqrt{9x-3}}{x-8} = \dfrac{7\sqrt{69}}{138}$

**Problema 2** Calcular las siguientes derivadas:

1. $y = e^{x^3+3x^2-x+3}$
2. $y = \ln(5x^4+2x-3)$
3. $y = (7x^2-3x+8)^{28}$
4. $y = (x^2-3x+9)(2x^3-3x^2-x+2)$
5. $y = \frac{x^2+3x-7}{2x+7}$
6. $y = \ln\frac{x^2+5x-1}{2x^2+7}$
7. $y = e^{5x^3-1}\cdot(x^3+4)$
8. $y = \frac{e^{x^2-1}}{x^3+2}$

**Solución:**

1. $y = e^{x^3+3x^2-x+3} \Longrightarrow y' = (3x^2+6x-1)e^{x^3+3x^2-x+3}$
2. $y = \ln(5x^4+2x-3) \Longrightarrow y' = \dfrac{20x^3+2}{5x^4+2x-3}$
3. $y = (7x^2-3x+8)^{28} \Longrightarrow y' = 28(7x^2-3x+8)^{27}(14x-3)$
4. $y = (x^2-3x+9)(2x^3-3x^2-x+2) \Longrightarrow y' = (2x-3)(2x^3-3x^2-x+2)+(x^2-3x+9)(6x^2-6x-1)$
5. $y = \dfrac{x^2+3x-7}{2x+7} \Longrightarrow y' = \dfrac{(2x+3)(2x+7)-(x^2+3x-7)2}{(2x+7)^2}$
6. $y = \ln\frac{x^2+5x-1}{2x^2+7} = \ln(x^2+5x-1)-\ln(2x^2+7) \Longrightarrow y' = \dfrac{2x+5}{x^2+5x-1} - \dfrac{4x}{2x^2+7}$
7. $y = e^{5x^3-1}\cdot(x^3+4) \Longrightarrow y' = (15x^2)e^{5x^3-1}(x^3+4)+e^{5x^3-1}(3x^2)$
8. $y = \frac{e^{x^2-1}}{x^3+2} \Longrightarrow y' = \dfrac{2xe^{x^2-1}(x^3+2)-e^{x^2-1}(3x^2)}{(x^3+2)^2}$

**Problema 3** Calcular

1. las rectas tangente y normal a la siguiente función: $f(x) = \dfrac{x^2+5}{x^2-2}$ en el punto $x = 2$.

2. las rectas tangente y normal a la siguiente función: $f(x) = 7e^{2x-4}$ en el punto $x = 2$.

**Solución:**

1. $b = f(a) \Longrightarrow b = f(2) = 9/2$ e $y - b = m(x - a)$

$$f'(x) = -\frac{14x}{(x^2-2)^2} \Longrightarrow m = f'(2) = -7$$

   Recta Tangente: $y + \frac{9}{2} = -7(x - 2)$

   Recta Normal: $y + \frac{9}{2} = \frac{1}{7}(x - 2)$

2. $b = f(a) \Longrightarrow b = f(2) = 2$ e $y - b = m(x - a)$

$$f'(x) = 14e^{2x-4} \Longrightarrow m = f'(2) = 14$$

   Recta Tangente: $y - 2 = 14(x - 2)$

   Recta Normal: $y - 2 = -\frac{1}{14}(x - 2)$