Examen de Matemáticas 1º de Bachillerato CN

Marzo 2021

Problema 1 Calcular las siguientes integrales:

a) $\displaystyle\int \frac{3x^2}{1+5x^3}\,dx$

b) $\displaystyle\int 5x(2x^2-3)^{22}\,dx$

c) $\displaystyle\int \frac{7x^2\cos x - 5x^2e^x + 4x - 1}{x^2}\,dx$

d) $\displaystyle\int \frac{5x^3 - 3\sqrt[5]{x^2} + 7x}{x^2}\,dx$

e) $\displaystyle\int \frac{4x^3\sin(x^2-1) + 7x^3e^{5x^2-3} - 8x + 2}{x^2}\,dx$

f) $\displaystyle\int \frac{-5}{1+x^2}\,dx$

Solución:

a) $\displaystyle\int \frac{3x^2}{1+5x^3}\,dx = \frac{1}{5}\ln|1+5x^3| + C$

b) $\displaystyle\int 5x(2x^2-3)^{22}\,dx = \frac{5(2x^2-3)^{23}}{92} + C$

c) $\displaystyle\int \frac{7x^2\cos x - 5x^2e^x + 4x - 1}{x^2}\,dx = 7\sin x - 5e^x + 4\ln|x| + \frac{1}{x} + C$

d) $\displaystyle\int \frac{5x^3 - 3\sqrt[5]{x^2} + 7x}{x^2}\,dx = \frac{5x^2}{2} + \frac{5}{x^{3/5}} + 7\ln|x| + C$

e) $\displaystyle\int \frac{4x^3\sin(x^2-1) + 7x^3e^{5x^2-3} - 8x + 2}{x^2}\,dx = -2\cos(x^2+1) + \frac{7e^{5x^2+9}}{10} - \frac{2}{x} - 8\ln|x| + C$

f) $\displaystyle\int \frac{-5}{1+x^2}\,dx = -5\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

a) $y = \ln\sqrt[3]{\dfrac{x^5\cos(x^2-5)}{e^{x^2+7}\sin x}}$

b) $y = (\sin x)^{x^5+1}$

c) $y = \dfrac{\arctan(x^4+3)(3x-1)}{x^2-1}$

d) $y = \csc(3x+1)^2\sec^2(x^2+1)$

e) $y = 7^{\cos^2 x - \sin x} \log_5(3x^2 - \cos x)$

f) $y = (\sqrt{x^2+5})^{\arctan x}$

Solución:

a) $y = \ln \sqrt[3]{\dfrac{x^5 \cos(x^2-5)}{e^{x^2+7} \sin x}} = \frac{1}{3}\left(5 \ln x + \ln \cos(x^2-5) - (x^2+7)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{3}\left(\frac{5}{x} + \frac{-2x\sin(x^2-5)}{\cos(x^2-5)} - 2x - \frac{\cos x}{\sin x}\right)$$

b) $y = (\sin x)^{x^5+1} \Longrightarrow y' = (\sin x)^{x^5+1}\left(5x^4 \ln \sin x + (x^5+1)\frac{\cos x}{\sin x}\right)$

c) $y = \dfrac{\arctan(x^4+3)(3x-1)}{x^2-1} \Longrightarrow$
$y' = \frac{\left(\frac{4x^3}{1+(x^4+3)^2}(3x-1)+3\arctan(x^4+3)\right)(x^2-1)-\arctan(x^4+3)(3x-1)2x}{(x^2-1)^2}$

d) $y = \csc(3x+1)^2 \sec^2(x^2+1) \Longrightarrow y' = -3(3x+1)\csc(3x+1)^2 \cot(3x+1)^2 \sec^2(x^2+1) + \csc(3x+1)^2 2\sec(x^2+1) 2x \sec(x^2+1)\tan(x^2+1)$

e) $y = 7^{\cos^2 x - \sin x}\log_5(3x^2 - \cos x) \Longrightarrow y' = (2\cos x(-\sin x) - \cos x)7^{\cos^2 x - \sin x}\ln 7 \log_5(3x^2 - \cos x) + 7^{\cos^2 x - \sin x}\frac{6x+\sin x}{(3x^2-\cos x)\ln 5}$

f) $y = (\sqrt{x^2+5})^{\arctan x} \Longrightarrow y' = (\sqrt{x^2+5})^{\arctan x}\left(\frac{1}{1+x^2}\ln\sqrt{x^2+5} + \arctan x \frac{\frac{2x}{2\sqrt{x^2+5}}}{\sqrt{x^2+5}}\right)$

Problema 3 Calcular los siguientes límites:

a) $\displaystyle \lim_{x \longrightarrow \infty}\left(\sqrt{2x^2+6x-3} - \sqrt{2x^2+9x+1}\right)$

b) $\displaystyle \lim_{x \longrightarrow 1}\frac{2x^4 - 7x^3 + 6x^2 + 2x - 3}{x^3 - 6x^2 + 6x - 1}$

c) $\displaystyle \lim_{x \longrightarrow 7}\frac{\sqrt{x^2+4} - \sqrt{8x-3}}{x-7}$

d) $\displaystyle \lim_{x \longrightarrow \infty}\left(\frac{7x^2 - 3x + 3}{7x^2 - 1}\right)^{6x}$

e) $\displaystyle \lim_{x \longrightarrow \infty}\frac{e^{6x^2-2}}{7x+1}$

f) $\displaystyle \lim_{x \longrightarrow \infty}\frac{e^{5x+3} - 6}{e^{7x+1} - 2}$

g) $\displaystyle \lim_{x \longrightarrow 0}\frac{\sin^2 x - 8x}{3x\cos x}$

Solución:

a) $\displaystyle \lim_{x \longrightarrow \infty}\left(\sqrt{2x^2+6x-3} - \sqrt{2x^2+9x+1}\right) = -\frac{3\sqrt{2}}{4}$

b) $\lim\limits_{x\longrightarrow 1} \dfrac{2x^4-7x^3+6x^2+2x-3}{x^3-6x^2+6x-1} = -\dfrac{1}{3}$

c) $\lim\limits_{x\longrightarrow 7} \dfrac{\sqrt{x^2+4}-\sqrt{8x-3}}{x-7} = \dfrac{3\sqrt{53}}{53}$

d) $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{7x^2-3x+3}{7x^2-1}\right)^{6x} = e^{-18/7}$

e) $\lim\limits_{x\longrightarrow\infty} \dfrac{e^{6x^2-2}}{7x+1} = \infty$

f) $\lim\limits_{x\longrightarrow\infty} \dfrac{e^{5x+3}-6}{e^{7x+1}-2} = 0$

g) $\lim\limits_{x\longrightarrow 0} \dfrac{\sin^2 x-8x}{3x\cos x} = -\dfrac{8}{3}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

a) $f(x) = \dfrac{5x-1}{x-1}$ en el punto $x = 2$.

b) $f(x) = (x+4)e^{x-1}$ en el punto $x = 1$.

Solución:

a) $b = f(a) \Longrightarrow b = f(2) = 9$ e $y - b = m(x-a)$

$$f'(x) = -\frac{4}{(x-1)^2} \Longrightarrow m = f'(2) = -4$$

Recta Tangente: $y - 9 = -4(x-2)$

Recta Normal: $y - 9 = \frac{1}{4}(x-2)$

b) $b = f(a) \Longrightarrow b = f(1) = 5$ e $y - b = m(x-a)$

$$f'(x) = (x+5)e^{x-1} \Longrightarrow m = f'(1) = 6$$

Recta Tangente: $y - 5 = 6(x-1)$

Recta Normal: $y - 5 = -\dfrac{1}{6}(x-1)$