Examen de Matemáticas 1º de Bachillerato CN

Marzo 2020

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{x^2-8x-1}-\sqrt{x^2+3x-3}\right)$
2. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7}$
3. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^3-8x^2-2x+3}{4x^3-x^2-6x+3}$
4. $\lim\limits_{x\longrightarrow 0}\dfrac{2\cos x+xe^x-2}{x\cos x}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x}+3x^2-1}{e^{2x}+7x+3}$
6. $\lim\limits_{x\longrightarrow 0}\dfrac{x\cos x+xe^x}{\cos x-e^x}$
7. $\lim\limits_{x\longrightarrow 0}\dfrac{\ln(x^3+1)}{\ln(x^2+1)}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{x^2-8x-1}-\sqrt{x^2+3x-3}\right)=-\dfrac{11}{2}$
2. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7}=\dfrac{\sqrt{51}}{17}$
3. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^3-8x^2-2x+3}{4x^3-x^2-6x+3}=\dfrac{3}{4}$
4. $\lim\limits_{x\longrightarrow 0}\dfrac{2\cos x+xe^x-2}{x\cos x}=1$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{e^{2x}+3x^2-1}{e^{2x}+7x+3}=1$
6. $\lim\limits_{x\longrightarrow 0}\dfrac{x\cos x+xe^x}{\cos x-e^x}=-2$
7. $\lim\limits_{x\longrightarrow 0}\dfrac{\ln(x^3+1)}{\ln(x^2+1)}=0$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln \sqrt[5]{\dfrac{x^2 \cos^3(5x)}{e^{2x} \sin x^2}}$
2. $y = (4x^3 - 1)^{\sin(7x)}$
3. $y = (\arccos x)^{3x^2+1}$
4. $y = \log_5 \dfrac{3x^2 - 1}{\sqrt{x^2 + 3}}$
5. $y = \sqrt[7]{\dfrac{x^2 - 5}{\cos^2(2x)}}$
6. $y = \sec^2(x^2 + 2) \log_3(x^2 - 1)$
7. $y = 3^{\arctan(x^2+9)} \tan^2(x - 3)$

Solución:

1. $y = \ln \sqrt[5]{\dfrac{x^2 \cos^3(5x)}{e^{2x} \sin x^2}} = \frac{1}{5} \left(2 \ln x + 3 \ln \cos(5x) - (2x) \ln e - \ln(\sin x^2)\right) \Longrightarrow$

$$y' = \frac{1}{5} \left(\frac{2}{x} + 2 \frac{-5 \sin(5x)}{\cos(5x)} - 2 - \frac{2x \cos x^2}{\sin x^2} \right)$$

2. $y = (4x^3 - 1)^{\sin(7x)} \Longrightarrow y' = (4x^3 - 1)^{\sin(7x)} \left(7 \cos(7x) \ln(4x^3 - 1) + \sin(7x) \frac{12x^2}{4x^3 - 1}\right)$

3. $y = (\arccos x)^{3x^2+1} \Longrightarrow y' = (\arccos x)^{3x^2+1} \left(6x \ln(\arccos x) + (3x^2 + 1) \frac{\frac{1}{\sqrt{1-x^2}}}{\arccos x} \right)$

4. $y = \log_5 \dfrac{3x^2 - 1}{\sqrt{x^2 + 3}} = \log_5(3x^2 - 1) - \dfrac{1}{2} \log_5(x^2 + 3) \Longrightarrow y' = \dfrac{6x}{(3x^2 - 1) \ln 5} - \dfrac{1}{2} \dfrac{2x}{(x^2 + 3) \ln 5}$

5. $y = \sqrt[7]{\dfrac{x^2 - 5}{\cos^2(2x)}} \Longrightarrow y' = \dfrac{1}{7} \left(\dfrac{x^2 - 5}{\cos^2(2x)} \right)^{-6/7} \left(\dfrac{2x \cos^2(2x) - (x^2 - 5)(2 \cos(2x)(-2 \sin(2x))}{\cos^4(2x)} \right)$

6. $y = \sec^2(x^2 + 2) \log_3(x^2 - 1) \Longrightarrow y' = 2x \sec^2(x^2 + 2) \tan(x^2 + 2) \log_3(x^2 - 1) + \sec^2(x^2 + 2) \dfrac{2x}{(x^2 - 1) \ln 3}$

7. $y = 3^{\arctan(x^2+9)} \tan^2(x - 3) \Longrightarrow y' = \dfrac{2x}{1 + (x^2 + 9)^2} 3^{\arctan(x^2+9)} \ln 3 \tan^2(x - 3) + 3^{\arctan(x^2+9)} 2 \tan(x - 3) \dfrac{1}{\cos^2(x - 3)}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{x^2 - 10}{2x + 5}$ en el punto $x = 0$.

2. $f(x) = (x^2 - 5)e^{2x}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(0) = -2$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(x^2 + 5x + 10)}{(2x + 5)^2} \Longrightarrow m = f'(0) = \frac{4}{5}$$

   Recta Tangente: $y + 2 = \frac{4}{5}x$

   Recta Normal: $y + 2 = -\frac{5}{4}x$

2. $b = f(a) \Longrightarrow b = f(0) = -5$ e $y - b = m(x - a)$

$$f'(x) = (x^2 + x - 5)e^{2x} \Longrightarrow m = f'(0) = -5$$

   Recta Tangente: $y + 5 = -5x$

   Recta Normal: $y + 5 = \dfrac{1}{5}x$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int 5xe^{6x^2-1}\, dx$

2. $\displaystyle\int \frac{7x}{5x^2 + 3}\, dx$

3. $\displaystyle\int 5x^2 \cos(2x^3 - 9)\, dx$

4. $\displaystyle\int \frac{3x}{1 + x^4}\, dx$

5. $\displaystyle\int \frac{7x^2 + 2x^2 \cos x - 3x^2 e^x + 6x}{x^2}\, dx$

6. $\displaystyle\int \frac{5x^5 - 3x^4 - 2\sqrt[5]{x^3} - 7x}{x^2}\, dx$

Solución:

1. $\displaystyle\int 5xe^{6x^2-1}\, dx = \frac{5}{12}e^{6x^2-1} + C$

2. $\displaystyle\int \frac{7x}{5x^2+3}\,dx = \frac{7}{10}\ln|5x^2+3| + C$

3. $\displaystyle\int 5x^2\cos(2x^3-9)\,dx = \frac{5}{6}\sin(2x^3-9) + C$

4. $\displaystyle\int \frac{3x}{1+x^4}\,dx = \frac{3}{2}\arctan x^2 + C$

5. $\displaystyle\int \frac{7x^2+2x^2\cos x-3x^2e^x+6x}{x^2}\,dx = 7x+2\sin x-3e^x+6\ln|x| + C$

6. $\displaystyle\int \frac{5x^5-3x^4-2\sqrt[5]{x^3}-7x}{x^2}\,dx = \frac{5x^4}{4}-x^3+5x^{-2/5}-7\ln|x| + C$