Examen de Matemáticas 1º de Bachillerato CN

Diciembre 2019

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty} \dfrac{3x^4 - x^2 + x + 1}{5x^4 - 2x - 1}$

2. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2 + 8x + 8}{5x^2 + 6x - 1}\right)^{x^2+9}$

3. $\lim_{x\longrightarrow\infty} \left(\dfrac{7x^2 - x + 3}{7x^2 - 1}\right)^{2x}$

4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{6x^4 + 3x^3 - 5x^2 - x + 1}}{5x^2 - x + 9}$

5. $\lim_{x\longrightarrow 1} \dfrac{6x^4 + 5x^3 - 11x^3 + 2x - 2}{x^3 - 4x^2 + 2x + 1}$

6. $\lim_{x\longrightarrow 2} \dfrac{5x^3 - 8x^2 - 5x + 2}{3x^3 - 5x^2 - 3x + 2}$

7. $\lim_{x\longrightarrow 7} \dfrac{\sqrt{x^2 + 2} - \sqrt{8x - 5}}{x - 7}$

8. $\lim_{x\longrightarrow 5} \dfrac{\sqrt{x^2 + 7} - \sqrt{7x - 3}}{x - 5}$

Solución:

1. $\lim_{x\longrightarrow\infty} \dfrac{3x^4 - x^2 + x + 1}{5x^4 - 2x - 1} = \dfrac{3}{5}$

2. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2 + 8x + 8}{5x^2 + 6x - 1}\right)^{x^2+9} = 0$

3. $\lim_{x\longrightarrow\infty} \left(\dfrac{7x^2 - x + 3}{7x^2 - 1}\right)^{2x} = e^{-2/7}$

4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{6x^4 + 3x^3 - 5x^2 - x + 1}}{5x^2 - x + 9} = \dfrac{\sqrt{6}}{5}$

5. $\lim_{x\longrightarrow 1} \dfrac{6x^4 + 5x^3 - 11x^3 + 2x - 2}{x^3 - 4x^2 + 2x + 1} = -\dfrac{8}{3}$

6. $\lim_{x\longrightarrow 2} \dfrac{5x^3 - 8x^2 - 5x + 2}{3x^3 - 5x^2 - 3x + 2} = \dfrac{23}{13}$

7. $\lim\limits_{x\longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7} = \dfrac{\sqrt{51}}{17}$

8. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{x^2+7}-\sqrt{7x-3}}{x-5} = \dfrac{3\sqrt{2}}{16}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3+x^2-2x-1}$
2. $y = \ln(3x^3-7)$
3. $y = (x^2+2x-1)^{32}$
4. $y = (2x^2-2x+3)(x^3-x^2-1)$
5. $y = \dfrac{x^2-5}{7x+3}$
6. $y = \ln \frac{x^2+4x-1}{x^2-3x-1}$
7. $y = (x^2+2)^{\sin x}$
8. $y = \arctan(x^2-3x-1)$
9. $y = \sqrt{3x^2+8x-3}$

Solución:

1. $y = e^{5x^3+x^2-2x-1} \Longrightarrow y' = (15x^2+2x-2)e^{5x^3+x^2-2x-1}$
2. $y = \ln(3x^3-7) \Longrightarrow y' = \dfrac{9x^2}{3x^3-7}$
3. $y = (x^2+2x-1)^{32} \Longrightarrow y' = 32(x^2+2x-1)^{31}(2x+2)$
4. $y = (2x^2-2x+3)(x^3-x^2-1) \Longrightarrow y' = (4x-2)(x^3-x^2-1)+(2x^2-2x+3)(3x^2-2x)$
5. $y = \dfrac{x^2-5}{7x+3} \Longrightarrow y' = \dfrac{(2x)(7x+3)-(x^2-5)7}{(7x+3)^2}$
6. $y = \ln \frac{x^2+4x-1}{x^2-3x-1} = \ln(x^2+4x-1)-\ln(x^2-3x-1) \Longrightarrow y' = \dfrac{2x+4}{x^2+4x-1} - \dfrac{2x-3}{x^2-3x-1}$
7. $y = (x^2+2)^{\sin x} \Longrightarrow y' = (x^2+2)^{\sin x}\left(\cos x \ln(x^2+2) + \sin x \frac{2x}{x^2+2}\right)$
8. $y = \arctan(x^2-3x-1) \Longrightarrow y' = \dfrac{2x-3}{1+(x^2-3x-1)^2}$

9. $y = \sqrt{3x^2 + 8x - 3} \Longrightarrow y' = \dfrac{6x + 8}{2\sqrt{3x^2 + 8x - 3}}$

Problema 3 Calcular las rectas tangente y normal a la siguiente funciones en el punto $x = 1$:

1. $f(x) = \dfrac{x^2 - 3x + 1}{x^2 + 2}$.

2. $f(x) = (x + 3)e^{2x-2}$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -1/3$ e $y - b = m(x - a)$

$$f'(x) = \frac{3x^2 + 2x - 6}{(x^2 + 2)^2} \Longrightarrow m = f'(1) = -\frac{1}{9}$$

Recta Tangente: $y + \frac{1}{3} = -\frac{1}{9}(x - 1)$

Recta Normal: $y + \frac{1}{3} = 9(x - 1)$

2. $b = f(a) \Longrightarrow b = f(1) = 4$ e $y - b = m(x - a)$

$$f'(x) = (2x + 7)e^{2x-2} \Longrightarrow m = f'(1) = 9$$

Recta Tangente: $y - 4 = 9(x - 1)$

Recta Normal: $y - 4 = -\dfrac{1}{9}(x - 1)$