Examen de Matemáticas 1º de Bachillerato CN

Marzo 2019

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \longrightarrow \infty} \left(\sqrt{x^2 + 7x - 1} - \sqrt{x^2 - 5x - 3} \right)$
2. $\lim_{x \longrightarrow 7} \frac{\sqrt{x^2 + 4} - \sqrt{6x + 11}}{x - 7}$
3. $\lim_{x \longrightarrow 1} \frac{5x^3 - 7x^2 + 7x - 5}{4x^3 + x^2 - 6x + 1}$
4. $\lim_{x \longrightarrow 0} \frac{\cos x + xe^x - 1}{x \cos x}$
5. $\lim_{x \longrightarrow \infty} \frac{e^{7x} + 3x}{e^{3x} + 7x}$
6. $\lim_{x \longrightarrow 0} \frac{x \sin x + xe^x}{\cos x - e^x}$
7. $\lim_{x \longrightarrow 0} \frac{\ln(x^2 + 8)}{\ln(3x^2 - 1)}$

Solución:

1. $\lim_{x \longrightarrow \infty} \left(\sqrt{x^2 + 7x - 1} - \sqrt{x^2 - 5x - 3} \right) = 6$
2. $\lim_{x \longrightarrow 7} \frac{\sqrt{x^2 + 2} - \sqrt{6x + 9}}{x - 7} = \frac{4\sqrt{51}}{51}$
3. $\lim_{x \longrightarrow 1} \frac{5x^3 - 7x^2 + 7x - 5}{4x^3 + x^2 - 6x + 1} = 1$
4. $\lim_{x \longrightarrow 0} \frac{\cos x + xe^x - 1}{x \cos x} = 1$
5. $\lim_{x \longrightarrow \infty} \frac{e^{7x} + 3x}{e^{3x} + 7x} = \infty$
6. $\lim_{x \longrightarrow 0} \frac{x \sin x + xe^x}{\cos x - e^x} = -1$
7. $\lim_{x \longrightarrow 0} \frac{\ln(x^2 + 8)}{\ln(3x^2 - 1)} = 1$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln \sqrt[5]{\dfrac{x^3 \cos^2(5x)}{e^{3x} \sin x^2}}$

2. $y = (4x^3 - 2)^{\sin(5x)}$

3. $y = (\arccos x)^{7x^2+3}$

4. $y = \log_7 \dfrac{5x^2 - 1}{\sqrt{x^2+1}}$

5. $y = \sqrt[6]{\dfrac{3x^2+2}{\cos^2(5x)}}$

6. $y = \sec^2(x^2 - 8) \log_3(x^2 + 5)$

7. $y = 2^{\arctan(x^2+5)} \tan^2(x - 1)$

Solución:

1. $y = \ln \sqrt[5]{\dfrac{x^3 \cos^2(5x)}{e^{3x} \sin x^2}} = \frac{1}{5}\left(3 \ln x + 2 \ln \cos(5x) - (3x) \ln e - \ln(\sin x^2)\right) \Longrightarrow$

$$y' = \frac{1}{5}\left(\frac{3}{x} + 2\frac{-5\sin(5x)}{\cos(5x)} - 3 - \frac{2x \cos x^2}{\sin x^2}\right)$$

2. $y = (4x^3-2)^{\sin(5x)} \Longrightarrow y' = (4x^3-2)^{\sin(5x)} \left(5\cos(5x)\ln(4x^3-2) + \sin(5x)\frac{12x^2}{4x^3-2}\right)$

3. $y = (\arccos x)^{7x^2+3} \Longrightarrow y' = (\arccos x)^{7x^2+3} \left(14x \ln(\arccos x) + (7x^2+3)\frac{\frac{1}{\sqrt{1-x^2}}}{\arccos x}\right)$

4. $y = \log_7 \dfrac{5x^2-1}{\sqrt{x^2+1}} = \log_7(5x^2-1) - \dfrac{1}{2}\log_7(x^2+1) \Longrightarrow y' = \dfrac{10x}{(5x^2-1)\ln 7} - \dfrac{1}{2}\dfrac{2x}{(x^2+1)\ln 7}$

5. $y = \sqrt[6]{\dfrac{3x^2+2}{\cos^2(5x)}} \Longrightarrow y' = \dfrac{1}{6}\left(\dfrac{3x^2+2}{\cos^2(5x)}\right)^{-5/6}\left(\dfrac{6x\cos^2(5x) - (3x^2+2)(2\cos(5x)(-5\sin(5x)))}{\cos^4(5x)}\right)$

6. $y = \sec^2(x^2-8)\log_3(x^2+5) \Longrightarrow y' = 4x\sec^2(x^2-8)\tan(x^2-8)\log_3(x^2+5) + \sec^2(x^2-8)\dfrac{2x}{(x^2+5)\ln 3}$

7. $y = 2^{\arctan(x^2+5)}\tan^2(x-1) \Longrightarrow y' = \dfrac{2x}{1+(x^2+5)^2}2^{\arctan(x^2+5)}\ln 2 \tan^2(x-1) + 2^{\arctan(x^2+5)} 2\tan(x-1)\dfrac{1}{\cos^2(x-1)}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{x^2 - 9}{5x + 2}$ en el punto $x = 0$.
2. $f(x) = (x^2 - 3)e^{3x}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(0) = -9/2$ e $y - b = m(x - a)$

$$f'(x) = \frac{5x^2 + 4x + 45}{(5x + 2)^2} \Longrightarrow m = f'(0) = \frac{45}{4}$$

   Recta Tangente: $y + \frac{9}{2} = \frac{45}{4}x$

   Recta Normal: $y + \frac{9}{2} = -\frac{4}{45}x$

2. $b = f(a) \Longrightarrow b = f(0) = -3$ e $y - b = m(x - a)$

$$f'(x) = (3x^2 + 2x - 9)e^{3x} \Longrightarrow m = f'(0) = -9$$

   Recta Tangente: $y + 3 = -9x$

   Recta Normal: $y + 3 = \dfrac{1}{9}x$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int 5xe^{7x^2+2}\,dx$
2. $\displaystyle\int \frac{3x}{2x^2 + 1}\,dx$
3. $\displaystyle\int 8x^2\cos(7x^3 - 1)\,dx$
4. $\displaystyle\int \frac{7x}{1 + x^4}\,dx$
5. $\displaystyle\int \frac{5x^2 + 4x^2\cos x - 2x^2e^x + 7x}{x^2}\,dx$
6. $\displaystyle\int \frac{2x^5 - 5x^4 + 3\sqrt[7]{x^3} - 2x}{x^2}\,dx$

Solución:

1. $\displaystyle\int 5xe^{7x^2+2}\,dx = \frac{5}{14}e^{7x^2+2} + C$

2. $\displaystyle\int \frac{3x}{2x^2+1}\,dx = \frac{3}{4}\ln|2x^2+1| + C$

3. $\displaystyle\int 8x^2\cos(7x^3-1)\,dx = \frac{8}{21}\sin(7x^3-1) + C$

4. $\displaystyle\int \frac{7x}{1+x^4}\,dx = \frac{7}{2}\arctan x^2 + C$

5. $\displaystyle\int \frac{5x^2+4x^2\cos x-2x^2e^x+7x}{x^2}\,dx = 5x + 4\sin x - 2e^x + 7\ln|x| + C$

6. $\displaystyle\int \frac{2x^5-5x^4+3\sqrt[7]{x^3}-2x}{x^2}\,dx = \frac{x^4}{2} - \frac{5x^3}{3} - \frac{21x^{-4/7}}{4} - 2\ln|x| + C$