Examen de Matemáticas 1º de Bachillerato CN

Marzo 2019

Problema 1 Calcular las siguientes integrales:

1. $\displaystyle\int \frac{2x^2}{1+7x^3}\,dx$

2. $\displaystyle\int 5x(3x^2+7)^{19}\,dx$

3. $\displaystyle\int \frac{3x^2\cos x - 8x^2e^x + 6x - 2}{x^2}\,dx$

4. $\displaystyle\int \frac{8x^3 - 3\sqrt[5]{x^2} + 7x}{x^2}\,dx$

5. $\displaystyle\int \frac{5x^3\sin(x^2+1) + 7x^3e^{5x^2+9} - x + 9}{x^2}\,dx$

6. $\displaystyle\int \frac{7}{1+x^2}\,dx$

Solución:

1. $\displaystyle\int \frac{2x^2}{1+7x^3}\,dx = \frac{2}{21}\ln|1+7x^3| + C$

2. $\displaystyle\int 5x(3x^2+7)^19\,dx = \frac{(3x^2+7)^20}{24} + C$

3. $\displaystyle\int \frac{3x^2\cos x - 8x^2e^x + 6x - 2}{x^2}\,dx = 3\sin x - 8e^x + 6\ln|x| + \frac{2}{x} + C$

4. $\displaystyle\int \frac{8x^3 - 3\sqrt[5]{x^2} + 7x}{x^2}\,dx = 4x^2 + 5x^{-3/5} + 7\ln|x| + C$

5. $\displaystyle\int \frac{5x^3\sin(x^2+1) + 7x^3e^{5x^2+9} - x + 9}{x^2}\,dx = -\frac{5\cos(x^2+1)}{2} + \frac{7e^{5x^2+9}}{10} - \frac{9}{x} - \ln|x| + C$

6. $\displaystyle\int \frac{7}{1+x^2}\,dx = 7\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln\sqrt[5]{\dfrac{x^5\cos(x^2+7)}{e^{x^2+1}\sin x}}$

2. $y = (\sin x)^{x^5+9}$

3. $y = \dfrac{\arctan(x^4+7)(5x-3)}{x^2-3}$

4. $y = \csc(2x-1)^2 \sec^2(x^2+7)$

5. $y = 9^{\cos^2 x - \sin x} \log_5(3x^2 - \cos x)$

6. $y = (\sqrt{5x^2-8})^{\arctan x}$

Solución:

1. $y = \ln \sqrt[5]{\dfrac{x^5\cos(x^2+7)}{e^{x^2+1}\sin x}} = \frac{1}{5}\left(5\ln x + \ln\cos(x^3+7) - (x^2+1)\ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{5}\left(\frac{5}{x} + \frac{-3x^2\sin(x^3+7)}{\cos(x^3+7)} - \frac{2x}{x^2+1} - \frac{\cos x}{\sin x}\right)$$

2. $y = (\sin x)^{x^5+9} \Longrightarrow y' = (\sin x)^{x^5+9}\left(5x^4\ln\sin x + (x^5+9)\frac{\cos x}{\sin x}\right)$

3. $y = \dfrac{\arctan(x^4+7)(5x-3)}{x^2-3} \Longrightarrow$
$y' = \dfrac{\left(\frac{4x^3}{1+(x^4+7)^2}(5x-3)+5\arctan(x^4+7)\right)(x^2-3)-\arctan(x^4+7)(5x-3)2x}{(x^2-3)^2}$

4. $y = \csc(2x-1)^2\sec^2(x^2+7) \Longrightarrow y' = -2(2x-1)\csc(5x-3)^2\cot(2x-1)^2\sec^2(x^2+7) + \csc(2x-1)^2 2\sec(x^2+7)2x\sec(x^2+7)\tan(x^2+7)$

5. $y = 9^{\cos^2 x-\sin x}\log_5(3x^2-\cos x) \Longrightarrow y' = (2\cos x(-\sin x)-\cos x)9^{\cos^2 x-\sin x}\ln 9\log_5(3x^2-\cos x) + 9^{\cos^2 x-\sin x}\frac{6x+\sin x}{(3x^2-\cos x)\ln 5}$

6. $y = (\sqrt{5x^2-8})^{\arctan x} \Longrightarrow y' = (\sqrt{5x^2-8})^{\arctan x}\left(\frac{1}{1+x^2}\ln\sqrt{5x^2-8} + \arctan x\frac{\frac{10x}{2\sqrt{5x^2-8}}}{\sqrt{5x^2-8}}\right)$

Problema 3 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{7x^2-5x+1} - \sqrt{7x^2+4x-2}\right)$

2. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4-8x^2-3x+4}{6x^5+2x-8}$

3. $\lim\limits_{x\longrightarrow 7}\dfrac{\sqrt{x^2-6}-\sqrt{6x+1}}{x-7}$

4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{2x^2-5x+3}{2x^2-7}\right)^{2x}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{5x^2+7}}{8x-1}$

6. $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{3x-1}+8}{e^{9x-2}-3}$

7. $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x - 8x}{6x \cos x}$

Solución:

1. $\lim\limits_{x \longrightarrow \infty} \left(\sqrt{7x^2-5x+1} - \sqrt{7x^2+4x-2} \right) = -\dfrac{9\sqrt{7}}{14}$

2. $\lim\limits_{x \longrightarrow 1} \dfrac{7x^4-8x^2-3x+4}{6x^5+2x-8} = \dfrac{9}{32}$

3. $\lim\limits_{x \longrightarrow 7} \dfrac{\sqrt{x^2-6}-\sqrt{6x+1}}{x-7} = \dfrac{4\sqrt{43}}{43}$

4. $\lim\limits_{x \longrightarrow \infty} \left(\dfrac{2x^2-5x+3}{2x^2-7} \right)^{2x} = e^{-5}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{5x^2+7}}{8x-1} = \infty$

6. $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{3x-1}+8}{e^{3x-2}-3} = e$

7. $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x - 8x}{6x \cos x} = -\dfrac{4}{3}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{8x+5}{x-1}$ en el punto $x = 2$.

2. $f(x) = (x+6)e^{x+1}$ en el punto $x = -1$.

Solución:

1. $b = f(a) \Longrightarrow b = f(2) = 21$ e $y - b = m(x-a)$

$$f'(x) = -\frac{13}{(x-1)^2} \Longrightarrow m = f'(2) = -13$$

Recta Tangente: $y - 21 = -13(x-2)$

Recta Normal: $y - 21 = \frac{1}{13}(x-2)$

2. $b = f(a) \Longrightarrow b = f(-1) = 5$ e $y - b = m(x - a)$

$$f'(x) = (x+7)e^{x+1} \Longrightarrow m = f'(-1) = 6$$

Recta Tangente: $y - 5 = 6(x + 1)$

Recta Normal: $y - 5 = -\dfrac{1}{6}(x + 1)$