Examen de Matemáticas 1º de Bachillerato CN

Marzo 2018

Problema 1 Calcular las siguientes integrales:

1. $\displaystyle\int \frac{7x^2}{1+5x^3}\,dx$

2. $\displaystyle\int 5x(x^2+9)^16\,dx$

3. $\displaystyle\int \frac{7x^2\cos x - 5x^2e^x + 3x - 1}{x^2}\,dx$

4. $\displaystyle\int \frac{7x^3 - 2\sqrt[5]{x^2} + 6x}{x^2}\,dx$

5. $\displaystyle\int \frac{4x^2\sin(x^2+3) + 8x^2e^{5x^2+1} - x + 9}{x^2}\,dx$

6. $\displaystyle\int \frac{9}{1+x^2}\,dx$

Solución:

1. $\displaystyle\int \frac{7x^2}{1+5x^3}\,dx = \frac{7}{15}\ln|1+5x^3| + C$

2. $\displaystyle\int 5x(x^2+9)^16\,dx = \frac{5(x^2+9)^17}{34} + C$

3. $\displaystyle\int \frac{7x^2\cos x - 5x^2e^x + 3x - 1}{x^2}\,dx = 7\sin x - 5e^x + 3\ln|x| - \frac{1}{x} + C$

4. $\displaystyle\int \frac{7x^3 - 2\sqrt[5]{x^2} + 6x}{x^2}\,dx = \frac{7x^2}{2} + \frac{10x^{-3/5}}{3} + 6\ln|x| + C$

5. $\displaystyle\int \frac{4x^2\sin(x^2+3) + 8x^2e^{5x^2+1} - x + 9}{x^2}\,dx = -2\cos(x^2+3) + \frac{4e^{5x^2+1}}{5} -$
$x + 9\ln|x| + C$

6. $\displaystyle\int \frac{9}{1+x^2}\,dx = 9\arctan x + C$

Problema 2 Calcular la primera derivada de las siguientes funciones:

1. $y = \ln\sqrt[5]{\dfrac{x^4\cos(x^3+1)}{e^{x^2+3}\sin x}}$

2. $y = (\sin x)^{x^3+2}$

3. $y = \dfrac{\arctan(x^4-5)(2x-1)}{x^3+9}$

4. $y = \csc(5x-3)^2 \sec^2(x^2-1)$

5. $y = 8^{\cos^2 x - \sin x} \log_3(x^2 - \cos x)$

6. $y = (\sqrt{x^3-2})^{\arctan x}$

Solución:

1. $y = \ln \sqrt[5]{\dfrac{x^4 \cos(x^3+1)}{e^{x^2+3} \sin x}} = \frac{1}{5}\left(4 \ln x + \ln \cos(x^3+1) - (x^2+3) \ln e - \ln(\sin x)\right) \Longrightarrow$

$$y' = \frac{1}{5}\left(\frac{4}{x} + \frac{-3x^2 \sin(x^3+1)}{\cos(x^3+1)} - \frac{2x}{x^2+3} - \frac{\cos x}{\sin x}\right)$$

2. $y = (\sin x)^{x^3+2} \Longrightarrow y' = (\sin x)^{x^3+2}\left(3x^2 \ln \sin x + (x^3+2)\frac{\cos x}{\sin x}\right)$

3. $y = \dfrac{\arctan(x^4-5)(2x-1)}{x^3+9} \Longrightarrow$
$y' = \dfrac{\left(\frac{4x^3}{1+(x^4-5)^2}(2x-1)+2\arctan(x^4-5)\right)(x^3+9)-\arctan(x^4-5)(2x-1)3x^2}{(x^3+9)^2}$

4. $y = \csc(5x-3)^2 \sec^2(x^2-1) \Longrightarrow 2(5x-3)5 \csc(5x-3)^2 \cot(5x-3)^2 \sec^2(x^2-1) + \csc(5x-3)^2 2 \sec(x^2-1) 2x \sec(x^2-1) \tan(x^2-1)$

5. $y = 8^{\cos^2 x - \sin x} \log_3(x^2 - \cos x) \Longrightarrow y' = (2 \cos x(-\sin x) - \cos x) 8^{\cos^2 x - \sin x} \ln 8 \log_3(x^2 - \cos x) + 8^{\cos^2 x - \sin x} \frac{2x + \sin x}{(x^2 - \cos x)\ln 3}$

6. $y = (\sqrt{x^3-2})^{\arctan x} \Longrightarrow y' = (\sqrt{x^3-2})^{\arctan x}\left(\frac{1}{1+x^2} \ln \sqrt{x^3-2} + \arctan x \frac{\frac{3x^2}{2\sqrt{x^3-2}}}{\sqrt{x^3-2}}\right)$

Problema 3 Calcular los siguientes límites:

1. $\lim\limits_{x \longrightarrow \infty}\left(\sqrt{5x^2-3x+3} - \sqrt{5x^2+4x-2}\right)$

2. $\lim\limits_{x \longrightarrow 1} \dfrac{7x^4 - 5x^2 + 3x - 5}{6x^5 - x - 5}$

3. $\lim\limits_{x \longrightarrow 7} \dfrac{\sqrt{x^2-8} - \sqrt{6x-1}}{x-7}$

4. $\lim\limits_{x \longrightarrow \infty}\left(\dfrac{2x^2-3x+2}{2x^2-1}\right)^{3x}$

5. $\lim\limits_{x \longrightarrow \infty} \dfrac{e^{2x^2+3}}{5x-1}$

6. $\lim\limits_{x\longrightarrow\infty} \dfrac{e^{2x-7}-15}{e^{2x+4}-3}$

7. $\lim\limits_{x\longrightarrow 0} \dfrac{\sin^2 x - 2x}{5x\cos x}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty} \left(\sqrt{5x^2-3x+3}-\sqrt{5x^2+4x-2}\right) = -\dfrac{7\sqrt{5}}{10}$

2. $\lim\limits_{x\longrightarrow 1} \dfrac{7x^4-5x^2+3x-5}{6x^5-x-5} = \dfrac{21}{29}$

3. $\lim\limits_{x\longrightarrow 7} \dfrac{\sqrt{x^2-8}-\sqrt{6x-1}}{x-7} = \dfrac{4\sqrt{41}}{41}$

4. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{2x^2-3x+2}{2x^2-1}\right)^{3x} = e^{-9/2}$

5. $\lim\limits_{x\longrightarrow\infty} \dfrac{e^{2x^2+3}}{5x-1} = \infty$

6. $\lim\limits_{x\longrightarrow\infty} \dfrac{e^{2x-7}-15}{e^{2x+4}-3} = e^{-11}$

7. $\lim\limits_{x\longrightarrow 0} \dfrac{\sin^2 x - 2x}{5x\cos x} = -\dfrac{2}{5}$

Problema 4 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{5x+2}{x-1}$ en el punto $x = 2$.

2. $f(x) = (x+5)e^{x+1}$ en el punto $x = -1$.

Solución:

1. $b = f(a) \Longrightarrow b = f(2) = 12$ e $y - b = m(x-a)$

$$f'(x) = -\frac{7}{(x-1)^2} \Longrightarrow m = f'(2) = -7$$

Recta Tangente: $y - 12 = -7(x-2)$

Recta Normal: $y - 12 = \frac{1}{7}(x-2)$

2. $b = f(a) \Longrightarrow b = f(-1) = 4$ e $y - b = m(x-a)$

$$f'(x) = (x+6)e^{x+1} \Longrightarrow m = f'(-1) = 5$$

Recta Tangente: $y - 4 = 5(x+1)$

Recta Normal: $y - 4 = -\dfrac{1}{5}(x+1)$