Examen de Matemáticas 1º de Bachillerato CS

Diciembre 2016

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \longrightarrow \infty} \dfrac{2x^3+7x^2+3x-1}{3x^3+5x^2-1}$

2. $\lim_{x \longrightarrow \infty} \left(\dfrac{2x^2+x+1}{3x^2-2x-1}\right)^{x^2+9}$

3. $\lim_{x \longrightarrow \infty} \left(\dfrac{x^2+5x+1}{x^2-1}\right)^{7x}$

4. $\lim_{x \longrightarrow \infty} \dfrac{\sqrt{4x^3-7x+1}}{3x^2-5}$

5. $\lim_{x \longrightarrow 1} \dfrac{7x^5+2x^4-9x^3+2x^2-4x+2}{2x^5-8x^4-10x^3+15x+1}$

6. $\lim_{x \longrightarrow 2} \dfrac{3x^3-6x^2+2x-4}{x^3-3x^2+x+2}$

7. $\lim_{x \longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{6x+9}}{x-7}$

8. $\lim_{x \longrightarrow 6} \dfrac{\sqrt{2x^2-1}-\sqrt{11x+5}}{x-6}$

Solución:

1. $\lim_{x \longrightarrow \infty} \dfrac{2x^3+7x^2+3x-1}{3x^3+5x^2-1} = \dfrac{2}{3}$

2. $\lim_{x \longrightarrow \infty} \left(\dfrac{2x^2+x+1}{3x^2-2x-1}\right)^{x^2+9} = 0$

3. $\lim_{x \longrightarrow \infty} \left(\dfrac{x^2+5x+1}{x^2-1}\right)^{7x} = e^{35}$

4. $\lim_{x \longrightarrow \infty} \dfrac{\sqrt{5x^3+3x+1}}{x^2+5} = 0$

5. $\lim_{x \longrightarrow 1} \dfrac{7x^5+2x^4-9x^3+2x^2-4x+2}{2x^5-8x^4-10x^3+15x+1} = -\dfrac{16}{37}$

6. $\lim_{x \longrightarrow 2} \dfrac{3x^3-6x^2+2x-4}{x^3-3x^2+x+2} = 14$

7. $\lim\limits_{x \longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{6x+9}}{x-7} = \dfrac{4\sqrt{51}}{51}$

8. $\lim\limits_{x \longrightarrow 6} \dfrac{\sqrt{2x^2-1}-\sqrt{11x+5}}{x-6} = \dfrac{13\sqrt{71}}{142}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3+4x^2-3x-2}$
2. $y = \ln(7x^4+3x-4)$
3. $y = (3x^2-5x+1)^{18}$
4. $y = (3x^2-5x+1)(x^3-3x^2+x-5)$
5. $y = \frac{x^2-8}{7x-1}$
6. $y = \ln\frac{3x^2+5}{5x^2-1}$
7. $y = 5^{3x^3+2x-1}$
8. $y = \log_5 x^2+2$

Solución:

1. $y = e^{5x^3+4x^2-3x-2} \Longrightarrow y' = (15x^2+8x-3)e^{5x^3+4x^2-3x-2}$
2. $y = \ln(7x^4+3x-4) \Longrightarrow y' = \dfrac{28x^3+3}{7x^4+3x-4}$
3. $y = (3x^2-5x+1)^{18} \Longrightarrow y' = 18(3x^2-5x+1)^{17}(6x-5)$
4. $y = (3x^2-5x+1)(x^3-3x^2+x-5) \Longrightarrow y' = (6x-5)(x^3-3x^2+x-5)+(3x^2-5x+1)(3x^2-6x+1)$
5. $y = \dfrac{x^2-8}{7x-1} \Longrightarrow y' = \dfrac{(2x)(7x-1)-(x^2-8)7}{(7x-1)^2}$
6. $y = \ln\dfrac{3x^2+5}{5x^2-1} = \ln(3x^2+5)-\ln(5x^2-1) \Longrightarrow y' = \dfrac{6x}{3x^2+5}-\dfrac{10x}{5x^2-1}$
7. $y = 5^{3x^3+2x-1} \Longrightarrow y' = (9x^2+2)e^{3x^3+2x-1}\ln 5$
8. $y = \log_5 x^2+2 \Longrightarrow y' = \dfrac{2x}{(x^2+2)\ln 5}$

Problema 3 Calcular

1. las rectas tangente y normal a la siguiente función: $f(x) = \dfrac{x^2+5}{x^2-1}$ en el punto $x=2$.

2. las rectas tangente y normal a la siguiente función: $f(x) = 3e^{x-2}$ en el punto $x = 2$.

Solución:

1. $b = f(a) \Longrightarrow b = f(2) = 3$ e $y - b = m(x - a)$

$$f'(x) = -\frac{12x}{(x^2-1)^2} \Longrightarrow m = f'(2) = -8/3$$

Recta Tangente: $y - 3 = -\frac{8}{3}(x - 2)$

Recta Normal: $y - 3 = \frac{3}{8}(x - 2)$

2. $b = f(a) \Longrightarrow b = f(2) = 3$ e $y - b = m(x - a)$

$$f'(x) = 3e^{x-2} \Longrightarrow m = f'(2) = 3$$

Recta Tangente: $y - 3 = 3(x - 2)$

Recta Normal: $y - 3 = -\frac{1}{3}(x - 2)$