Examen de Matemáticas 1º de Bachillerato CN

Diciembre 2016

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty} \dfrac{4x^3-7x^2+2x-3}{5x^3-x^2-5}$

2. $\lim_{x\longrightarrow\infty} \left(\dfrac{2x^2+3x-1}{3x^2-5x-2}\right)^{x^2-x+9}$

3. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2+5x-1}{x^2+5}\right)^{7x}$

4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{5x^4+x^3-2x^2+3x+1}}{x^2+x-5}$

5. $\lim_{x\longrightarrow 1} \dfrac{8x^5+2x^4-7x^3-5x^2-2x+4}{5x^5+4x^4-10x^3-x+2}$

6. $\lim_{x\longrightarrow 2} \dfrac{4x^3-5x^2-5x-2}{x^3-5x+2}$

7. $\lim_{x\longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7}$

8. $\lim_{x\longrightarrow 6} \dfrac{\sqrt{2x^2+1}-\sqrt{12x+1}}{x-6}$

Solución:

1. $\lim_{x\longrightarrow\infty} \dfrac{4x^3-7x^2+2x-3}{5x^3-x^2-5} = \dfrac{4}{5}$

2. $\lim_{x\longrightarrow\infty} \left(\dfrac{2x^2+3x-1}{3x^2-5x-2}\right)^{x^2-x+9} = 0$

3. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2+5x-1}{x^2+5}\right)^{7x} = e^{35}$

4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{5x^4+x^3-2x^2+3x+1}}{x^2+x-5} = \sqrt{5}$

5. $\lim_{x\longrightarrow 1} \dfrac{8x^5+2x^4-7x^3-5x^2-2x+4}{5x^5+4x^4-10x^3-x+2} = \dfrac{3}{2}$

6. $\lim_{x\longrightarrow 2} \dfrac{4x^3-5x^2-5x-2}{x^3-5x+2} = \dfrac{23}{7}$

7. $\lim_{x\longrightarrow 7} \dfrac{\sqrt{x^2+2}-\sqrt{8x-5}}{x-7} = \dfrac{\sqrt{51}}{17}$

8. $\lim_{x\longrightarrow 6} \dfrac{\sqrt{2x^2+1}-\sqrt{12x+1}}{x-6} = \dfrac{6\sqrt{73}}{73}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3-4x^2-6x-2}$
2. $y = \ln(2x^3+9)$
3. $y = (x^2-x+4)^{15}$
4. $y = (3x^2+2x-3)(2x^3-3x^2-5)$
5. $y = \dfrac{x^2-2}{5x+1}$
6. $y = \ln \frac{x^2-x-1}{x^2+2x-8}$
7. $y = (x^2-1)^{\cos x}$
8. $y = \arctan(x^2-5x+3)$
9. $y = \sqrt{7x^2+x-8}$

Solución:

1. $y = e^{5x^3-4x^2-6x-2} \Longrightarrow y' = (15x^2-8x-6)e^{5x^3-4x^2-6x-2}$
2. $y = \ln(2x^3+9) \Longrightarrow y' = \dfrac{6x^2}{2x^3+9}$
3. $y = (x^2-x+4)^{15} \Longrightarrow y' = 15(x^2-x+4)^{14}(2x-1)$
4. $y = (3x^2+2x-3)(2x^3-3x^2-5) \Longrightarrow y' = (6x+2)(2x^3-3x^2-5) + (3x^2+2x-3)(6x^2-6x)$
5. $y = \dfrac{x^2-2}{5x+1} \Longrightarrow y' = \dfrac{(2x)(5x+1)-(x^2+9)5}{(5x+1)^2}$
6. $y = \ln\dfrac{x^2-x-1}{x^2+2x-8} = \ln(x^2-x-1) - \ln(x^2+2x-8) \Longrightarrow y' = \dfrac{2x-1}{x^2-x-1} - \dfrac{2x+2}{x^2+2x-8}$
7. $y = (x^2-1)^{\cos x} \Longrightarrow y' = (x^2-1)^{\cos x}\left(-\sin x \ln(x^2-1) + \cos x \frac{2x}{x^2-1}\right)$
8. $y = \arctan(x^2-5x+3) \Longrightarrow y' = \dfrac{2x-5}{1+(x^2-5x+3)^2}$

9. $y = \sqrt{7x^2 + x - 8} \Longrightarrow y' = \dfrac{14x + 1}{2\sqrt{7x^2 + x - 8}}$

Problema 3 Calcular las rectas tangente y normal a la siguiente funciones en el punto $x = 1$:

1. $f(x) = \dfrac{x^2 + x - 5}{x^2 + 3}$.
2. $f(x) = (x - 3)e^{x-1}$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -3/4$ e $y - b = m(x - a)$

$$f'(x) = -\frac{x^2 - 16x - 3}{(x^2 + 3)^2} \Longrightarrow m = f'(1) = \frac{9}{8}$$

   Recta Tangente: $y + \frac{3}{4} = \frac{9}{8}(x - 1)$

   Recta Normal: $y + \frac{3}{4} = -\frac{8}{9}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(1) = -2$ e $y - b = m(x - a)$

$$f'(x) = (x - 2)e^{x-1} \Longrightarrow m = f'(1) = -1$$

   Recta Tangente: $y + 2 = -(x - 1)$

   Recta Normal: $y + 2 = (x - 1)$