Examen de Matemáticas 1º de Bachillerato CN
Diciembre 2015

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty} \frac{9x^3-5x^2+x+1}{3x^3-4x^2+5}$
2. $\lim_{x\longrightarrow\infty} \left(\frac{7x^2-2x+1}{3x^2+2x-2}\right)^{x^2-9}$
3. $\lim_{x\longrightarrow\infty} \left(\frac{x^2-2x+1}{x^2-5}\right)^{3x}$
4. $\lim_{x\longrightarrow\infty} \frac{\sqrt{5x^3+3x+1}}{x^2+5}$
5. $\lim_{x\longrightarrow 1} \frac{6x^5+2x^4-9x^3+x^2-2x+2}{3x^5+5x^4-10x^3+x+1}$
6. $\lim_{x\longrightarrow 2} \frac{3x^3-5x^2-x-2}{x^3-x^2-x-2}$
7. $\lim_{x\longrightarrow 3} \frac{\sqrt{5x^2+2}-\sqrt{15x+2}}{x-3}$
8. $\lim_{x\longrightarrow 5} \frac{\sqrt{x^2+2}-\sqrt{4x+7}}{x-5}$

Solución:

1. $\lim_{x\longrightarrow\infty} \frac{9x^3-5x^2+x+1}{3x^3-4x^2+5} = 3$
2. $\lim_{x\longrightarrow\infty} \left(\frac{7x^2-2x+1}{3x^2+2x-2}\right)^{x^2-9} = \infty$
3. $\lim_{x\longrightarrow\infty} \left(\frac{x^2-2x+1}{x^2-5}\right)^{3x} = e^{-6}$
4. $\lim_{x\longrightarrow\infty} \frac{\sqrt{5x^3+3x+1}}{x^2+5} = 0$
5. $\lim_{x\longrightarrow 1} \frac{6x^5+2x^4-9x^3+x^2-2x+2}{3x^5+5x^4-10x^3+x+1} = \frac{11}{6}$
6. $\lim_{x\longrightarrow 2} \frac{3x^3-5x^2-x-2}{x^3-x^2-x-2} = \frac{15}{7}$

7. $\lim\limits_{x\longrightarrow 3} \dfrac{\sqrt{5x^2+2}-\sqrt{15x+2}}{x-3} = \dfrac{15\sqrt{47}}{94}$

8. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{x^2+2}-\sqrt{4x+7}}{x-5} = \dfrac{\sqrt{3}}{3}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{4x^3-3x^2-2x-1}$
2. $y = \ln(5x^3+4)$
3. $y = (x^2+7x-1)^{16}$
4. $y = (x^2+x-3)(2x^3-x^2+5)$
5. $y = \dfrac{x^2+9}{5x+3}$
6. $y = \ln \frac{x^2-1}{x^2+8}$
7. $y = (x^2-8)^{\sin x}$
8. $y = \arctan(x^2-2x+1)$
9. $y = \sqrt{x^2-8}$

Solución:

1. $y = e^{4x^3-3x^2-2x-1} \Longrightarrow y' = (12x^2-6x-2)e^{4x^3-3x^2-2x-1}$
2. $y = \ln(5x^3+4) \Longrightarrow y' = \dfrac{15x^2}{5x^3+4}$
3. $y = (x^2+7x-1)^{16} \Longrightarrow y' = 16(x^2+7x-1)^{15}(2x+7)$
4. $y = (x^2+x-3)(2x^3-x^2+5) \Longrightarrow y' = (2x+1)(2x^3-x^2+5)+(x^2+x-3)(6x^2-2x)$
5. $y = \dfrac{x^2+9}{5x+3} \Longrightarrow y' = \dfrac{(2x)(5x+3)-(x^2+9)5}{(5x+3)^2}$
6. $y = \ln\dfrac{x^2-1}{x^2+8} = \ln(x^2-1)-\ln(x^2+8) \Longrightarrow y' = \dfrac{2x}{x^2-1} - \dfrac{2x}{x^2+8}$
7. $y = (x^2-8)^{\sin x} \Longrightarrow y' = (x^2-8)^{\sin x}\left(\cos x \ln(x^2-8) + \sin x \frac{2x}{x^2-8}\right)$
8. $y = \arctan(x^2-2x+1) \Longrightarrow y' = \dfrac{2x-2}{1+(x^2-2x+1)^2}$
9. $y = \sqrt{x^2-8} \Longrightarrow y' = \dfrac{2x}{2\sqrt{x^2-8}}$

Problema 3 Calcular

1. las rectas tangente y normal a la siguiente función: $f(x) = \dfrac{x^2-3}{x^2+1}$ en el punto $x = 1$.

2. la recta tangente a la función $f(x) = 7x^2 - 3x - 1$ que sea paralela a la recta $y = 11x + 7$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -1$ e $y - b = m(x - a)$

$$f'(x) = -\frac{8x}{(x^2+1)^2} \Longrightarrow m = f'(1) = 2$$

Recta Tangente: $y + 1 = 2(x - 1)$

Recta Normal: $y + 1 = -\frac{1}{2}(x - 1)$

2. $m = 1$, $f'(x) = 14x - 3 \Longrightarrow m = f'(a) = 14a - 3 = 11 = 1 \Longrightarrow a = 1$ $b = f(a) \Longrightarrow b = f(1) = 3$ e $y - b = m(x - a)$ Recta Tangente: $y - 3 = 11(x - 1)$