Examen de Matemáticas 1º de Bachillerato CS

Noviembre 2014

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty} \dfrac{8x^3+3x^2-5x+1}{4x^3-x^2-2}$
2. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2-x+3}{5x^2-x-2}\right)^{x^2+8}$
3. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2-x+3}{x^2+1}\right)^{x-1}$
4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{3x^2+x+3}}{x^2-4}$
5. $\lim_{x\longrightarrow 1} \dfrac{7x^5+2x^4-7x^3+x^2-2x-1}{4x^5+5x^4-12x^3+2x+1}$
6. $\lim_{x\longrightarrow 2} \dfrac{3x^3-4x^2-5x+2}{x^3+x^2-5x-2}$
7. $\lim_{x\longrightarrow 3} \dfrac{\sqrt{5x^2-2}-\sqrt{15x-2}}{x-3}$
8. $\lim_{x\longrightarrow 5} \dfrac{\sqrt{x^2-6}-\sqrt{3x+4}}{x-5}$

Solución:

1. $\lim_{x\longrightarrow\infty} \dfrac{8x^3+3x^2-5x+1}{4x^3-x^2-2} = 2$
2. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2-x+3}{5x^2-x-2}\right)^{x^2+8} = 0$
3. $\lim_{x\longrightarrow\infty} \left(\dfrac{x^2-x+3}{x^2+1}\right)^{x-1} = e^{-1}$
4. $\lim_{x\longrightarrow\infty} \dfrac{\sqrt{3x^2+x+3}}{x^2-4} = 0$
5. $\lim_{x\longrightarrow 1} \dfrac{7x^5+2x^4-7x^3+x^2-2x-1}{4x^5+5x^4-12x^3+2x+1} = \dfrac{11}{3}$
6. $\lim_{x\longrightarrow 2} \dfrac{3x^3-4x^2-5x+2}{x^3+x^2-5x-2} = \dfrac{15}{11}$

7. $\lim\limits_{x \longrightarrow 3} \dfrac{\sqrt{5x^2-2}-\sqrt{15x-2}}{x-3} = \dfrac{15\sqrt{43}}{86}$

8. $\lim\limits_{x \longrightarrow 5} \dfrac{\sqrt{x^2-6}-\sqrt{3x+4}}{x-5} = \dfrac{7\sqrt{19}}{38}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3+2x^2-x-1}$
2. $y = \ln(2x^3+1)$
3. $y = (x^2+5x-3)^{15}$
4. $y = (x^2-3x-1)(2x^3-x^2-2)$
5. $y = \frac{x^2+x+1}{5x+2}$
6. $y = \ln\frac{x^2+2}{x^2+3}$

Solución:

1. $y = e^{5x^3+2x^2-x-1} \Longrightarrow y' = (15x^2+4x-1)e^{5x^3+2x^2-x-1}$
2. $y = \ln(2x^3+1) \Longrightarrow y' = \dfrac{6x^2}{2x^3+1}$
3. $y = (x^2+5x-3)^{15} \Longrightarrow y' = 15(x^2+5x-3)^{14}(2x+5)$
4. $y = (x^2-3x-1)(2x^3-x^2-2) \Longrightarrow y' = (2x-3)(2x^3-x^2-2)+(x^2-3x-1)(6x^2-2x)$
5. $y = \dfrac{x^2+x+1}{5x+2} \Longrightarrow y' = \dfrac{(2x+1)(5x+2)-(x^2+x+1)5}{(5x+2)^2}$
6. $y = \ln\dfrac{x^2+2}{x^2+3} = \ln(x^2+2)-\ln(x^2+3) \Longrightarrow y' = \dfrac{2x}{x^2+2} - \dfrac{2x}{x^2+3}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{3x^2+4}{x^2-2}$ en el punto $x = 1$.
2. $f(x) = \dfrac{x^2+7}{2x-1}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -7$ e $y - b = m(x - a)$

$$f'(x) = -\frac{20x}{(x^2-2)^2} \Longrightarrow m = f'(1) = -20$$

Recta Tangente: $y + 7 = -20(x - 1)$

Recta Normal: $y + 7 = \frac{1}{20}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -7$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(x^2 - x - 7)}{(2x-1)^2} \Longrightarrow m = f'(0) = -14$$

Recta Tangente: $y + 7 = -14x$

Recta Normal: $y + 7 = \dfrac{1}{14}x$