Examen de Matemáticas 1º de Bachillerato CN

Junio 2015

Problema 1 Calcular las siguientes integrales:

1. $\displaystyle\int \frac{15x^2}{2x^3+9}\,dx$
2. $\displaystyle\int \frac{7x}{\cos^2(x^2+2)}\,dx$
3. $\displaystyle\int 7x^2e^{4x^3-1}\,dx$
4. $\displaystyle\int x(x^2+1)^{12}\,dx$
5. $\displaystyle\int \frac{x^2}{\sqrt{x^3-2}}\,dx$
6. $\displaystyle\int \frac{5x}{1+x^4}\,dx$
7. $\displaystyle\int \frac{x^4-5\sqrt[3]{x}+x}{x^2}\,dx$
8. $\displaystyle\int \frac{x^3+4}{x^2+3x-10}\,dx$
9. $\displaystyle\int \frac{x^2+1}{x^2-6x+9}\,dx$
10. $\displaystyle\int (x+1)\sin x\,dx$
11. $\displaystyle\int x^2e^x\,dx$
12. $\displaystyle\int x^4\ln x\,dx$
13. $\displaystyle\int e^{x+1}\cos x\,dx$

Solución:

1. $\displaystyle\int \frac{15x^2}{2x^3+9}\,dx = \frac{5}{3}\ln|2x^3+9| + C$
2. $\displaystyle\int \frac{7x}{\cos^2(x^2+2)}\,dx = \frac{7}{2}\tan(x^2+2) + C$

3. $\displaystyle\int 7x^2e^{4x^3-1}\,dx = \frac{7}{12}e^{4x^3-1} + C$

4. $\displaystyle\int x(x^2+1)^{12}\,dx = \frac{(x^2+1)^{13}}{26} + C$

5. $\displaystyle\int \frac{x^2}{\sqrt{x^3-2}}\,dx = \frac{2}{3}\sqrt{x^3-2} + C$

6. $\displaystyle\int \frac{5x}{1+x^4}\,dx = \frac{5}{2}\arctan x^2$

7. $\displaystyle\int \frac{x^4 - 5\sqrt[3]{x} + x}{x^2}\,dx = \frac{x^3}{3} + 3x^{-2/3} - \frac{1}{x} + C$

8. $\displaystyle\int \frac{x^3+4}{x^2+3x-10}\,dx = \frac{x^2}{2} - 3x + \frac{12}{7}\ln|x-2| + \frac{121}{7}\ln|x+5| + C$

9. $\displaystyle\int \frac{x^2+1}{x^2-6x+9}\,dx = x + 6\ln|x-3| - \frac{10}{x-3} + C$

10. $\displaystyle\int (x+1)\sin x\,dx = -(x+1)\cos x + \sin x + C$

11. $\displaystyle\int x^2e^x\,dx = e^x(x^2-2x+2) + C$

12. $\displaystyle\int x^4\ln x\,dx = \frac{x^5}{5}\ln x - \frac{x^5}{25} + C$

13. $\displaystyle\int e^{x+1}\cos x\,dx \frac{e^{x+1}(\sin x + \cos x)}{2} + C$

Problema 2 Calcular el área encerrada por las gráficas de las funciones $f(x) = 5x^2 + 9x - 12$ y $g(x) = 3x^2 + 3x + 8$. **Solución:**

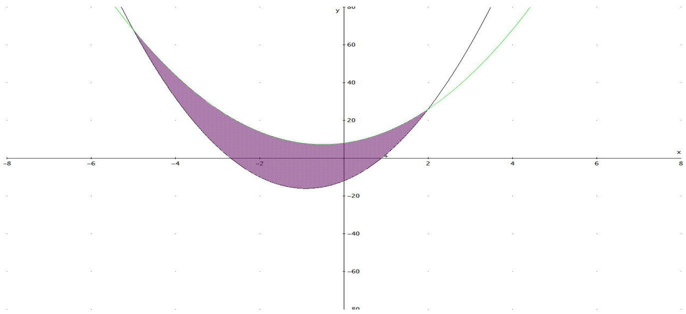
$$f(x) = g(x) \Longrightarrow 5x^2 + 9x - 12 = 3x^2 + 3x + 8 \Longrightarrow x = -5, \;\; x = 2$$

$$F(x) = \int (f(x) - g(x))\,dx = \int (2x^2 + 6x - 20)\,dx = \frac{2x^3}{3} + 3x^2 - 20x$$

$$S_1 = \int_{-5}^{2} (f(x) - g(x))\,dx = F(2) - f(-5) = -\frac{343}{3}\;u^2$$

Problema 3 Calcular el área encerrada por la gráfica de la función $f(x) = x^3 + 3x^2 - 10x$ el eje OX y las rectas $x = 0$ y $x = 3$. **Solución:**

$$f(x) = 0 \Longrightarrow x^3 + 3x^2 - 10x = 0 \Longrightarrow x = 0, \;\; x = -5, \;\; x = 2$$



Luego tenemos dos áreas. S_1 en $[0,2]$ y S_2 en $[2,3]$

$$F(x) = \int f(x)\,dx = \int (x^3 + 3x^2 - 10x)\,dx = \frac{x^4}{4} + x^3 - 5x^2$$

$$S_1 = \int_0^2 f(x)\,dx = F(2) - f(0) = -8 \quad S_2 = \int_2^3 f(x)\,dx = F(3) - f(2) = \frac{41}{4}$$

$$S = |S_1| + |S_2| = |-8| + \left|\frac{41}{4}\right| = \frac{73}{4}\ u^2$$