Examen de Matemáticas 1º de Bachillerato CN

Abril 2015

Problema 1 Calcular los siguientes límites:

1. $\lim_{x\longrightarrow\infty}\left(\sqrt{3x^2-5x+3}-\sqrt{3x^2+4x-2}\right)$
2. $\lim_{x\longrightarrow 1}\frac{7x^5-4x^3+4x-7}{6x^5-2x-4}$
3. $\lim_{x\longrightarrow 9}\frac{\sqrt{x^2-2}-\sqrt{8x+7}}{x-9}$
4. $\lim_{x\longrightarrow\infty}\left(\frac{2x^2-x+3}{2x^2-5}\right)^{5x}$
5. $\lim_{x\longrightarrow\infty}\frac{e^{3x^2-5}}{7x+1}$
6. $\lim_{x\longrightarrow\infty}\frac{e^{3x-1}-10}{e^{3x+1}-5}$
7. $\lim_{x\longrightarrow 0}\frac{2\sin^2 x-7x}{3x\cos x}$

Solución:

1. $\lim_{x\longrightarrow\infty}\left(\sqrt{3x^2-5x+3}-\sqrt{3x^2+4x-2}\right)=-\frac{3\sqrt{3}}{2}$
2. $\lim_{x\longrightarrow 1}\frac{7x^5-4x^3+4x-7}{6x^5-2x-4}=\frac{27}{28}$
3. $\lim_{x\longrightarrow 9}\frac{\sqrt{x^2-2}-\sqrt{8x+7}}{x-9}=\frac{5\sqrt{79}}{79}$
4. $\lim_{x\longrightarrow\infty}\left(\frac{2x^2-x+3}{2x^2-5}\right)^{5x}=e^{-5/2}$
5. $\lim_{x\longrightarrow\infty}\frac{e^{3x^2-5}}{7x+1}=\infty$
6. $\lim_{x\longrightarrow\infty}\frac{e^{3x-1}-10}{e^{3x+1}-5}=e^{-2}$
7. $\lim_{x\longrightarrow 0}\frac{2\sin^2 x-7x}{3x\cos x}=-\frac{7}{3}$

Problema 2 Calcular las siguientes derivadas:

1. $y = (9x^2 - 10)^{12}$
2. $y = \ln\left(\dfrac{8x-2}{\sin^2 x}\right)$
3. $y = (x-4)^5 \sec x$
4. $y = \dfrac{\cos^2 x}{x^2 - \sin x}$
5. $y = \sec(5x^3 + 2x + 1)^3$
6. $y = (\sin 2x)^{x^2-1}$

Solución:

1. $y = (9x^2 - 10)^{12} \Longrightarrow y' = 12(9x^2-10)^{11}(18x)$
2. $y = \ln\left(\dfrac{8x-2}{\sin^2 x}\right) \Longrightarrow y' = \dfrac{8}{8x-2} - \dfrac{2\cos x}{\sin x}$
3. $y = (x-4)^5 \sec x \Longrightarrow y' = 5(x-4)^4 \sec x + (x-4)^5 \sec x \tan x$
4. $y = \dfrac{\cos^2 x}{x^2 - \sin x} \Longrightarrow y' = \dfrac{-2\sin x \cos x \cdot (x^2 - \sin x) - (\cos^2 x)(2x - \cos x)}{(x^2 - \sin x)^2}$
5. $y = \sec(5x^3+2x+1)^3 \Longrightarrow y' = 3(15x^2+2)(5x^3+2x+1)^2 \tan(5x^3 + 2x+1)^3 \sec(5x^3+2x+1)^3$
6. $y = (\sin 2x)^{x^2-1} \Longrightarrow y' = (\sin 2x)^{x^2-1}\left(2x\ln(\sin 2x) + (x^2-1)\dfrac{2\cos 2x}{\sin 2x}\right)$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{8x+5}{x-3}$ en el punto $x = 2$.
2. $f(x) = (x+1)e^{2x+2}$ en el punto $x = -1$.

Solución:

1. $b = f(a) \Longrightarrow b = f(2) = -21$ e $y - b = m(x-a)$

   $$f'(x) = -\frac{29}{(x-3)^2} \Longrightarrow m = f'(2) = -29$$

   Recta Tangente: $y + 21 = -29(x-2)$
   Recta Normal: $y + 21 = \frac{1}{29}(x-2)$

2. $b = f(a) \Longrightarrow b = f(-1) = 0$ e $y - b = m(x - a)$

$$f'(x) = e^{2x+2} + 2e^{2x+2}(x+1) \Longrightarrow m = f'(-1) = 1$$

Recta Tangente: $y = x + 1 \Longrightarrow x - y + 1 = 0$
Recta Normal: $y = -(x+1) \Longrightarrow x + y - 1 = 0$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int (3x^2 + 7x - 2)\, dx$

2. $\displaystyle\int \left(\frac{5x^2 - 2\sqrt[4]{x} - 2}{x} - 8e^x \right) dx$

3. $\displaystyle\int \frac{3x}{1 + x^4}\, dx$

4. $\displaystyle\int 3xe^{7x^2+5}\, dx$

5. $\displaystyle\int \frac{8x}{3x^2 - 8}\, dx$

Solución:

1. $\displaystyle\int (3x^2 + 7x - 2)\, dx = x^3 + \frac{7x^2}{2} - 2x + C$

2. $\displaystyle\int \left(\frac{5x^2 - 2\sqrt[4]{x} - 2}{x} - 8e^x \right) dx = \frac{5x^2}{2} - 8x^{1/4} - 2\ln|x| - 8e^x + C$

3. $\displaystyle\int \frac{3x}{1 + x^4}\, dx = \frac{3\arctan x^2}{2} + C$

4. $\displaystyle\int 3xe^{7x^2+5}\, dx = \frac{3}{14}e^{7x^2+5} + C$

5. $\displaystyle\int \frac{8x}{3x^2 - 8}\, dx = \frac{4}{3}\ln|3x^2 - 8| + C$