Examen de Matemáticas 1º de Bachillerato CN

Diciembre 2014

Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{3x^2+x+1}-\sqrt{3x^2+2x-1}\right)$
2. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4+2x^3-3x^2-4x-2}{4x^5+x^2-2x-3}$
3. $\lim\limits_{x\longrightarrow 6}\dfrac{\sqrt{x^2-5}-\sqrt{5x+1}}{x-6}$
4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2-2x+1}{x^2}\right)^{x+1}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{7x^2-x+3}}{-x+5}$
6. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^2-\sqrt{x-3}+1}{4x^2+1}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty}\left(\sqrt{3x^2+x+1}-\sqrt{3x^2+2x-1}\right)=-\dfrac{\sqrt{3}}{6}$
2. $\lim\limits_{x\longrightarrow 1}\dfrac{7x^4+2x^3-3x^2-4x-2}{4x^5+x^2-2x-3}=\dfrac{6}{5}$
3. $\lim\limits_{x\longrightarrow 6}\dfrac{\sqrt{x^2-5}-\sqrt{5x+1}}{x-6}=\dfrac{7\sqrt{31}}{62}$
4. $\lim\limits_{x\longrightarrow\infty}\left(\dfrac{x^2-2x+1}{x^2}\right)^{x+1}=e^{-2}$
5. $\lim\limits_{x\longrightarrow\infty}\dfrac{\sqrt{7x^2-x+3}}{-x+5}=-\sqrt{7}$
6. $\lim\limits_{x\longrightarrow\infty}\dfrac{5x^2-\sqrt{x-3}+1}{4x^2+1}=\dfrac{5}{4}$

Problema 2 Calcular las siguientes derivadas:

1. $y=(2x^2+3x-1)^{16}$
2. $y=\ln\left(\dfrac{5x^2+2}{x^3-1}\right)$

3. $y = x^3 \sec x$

4. $y = \dfrac{\cos x}{5x^2 - 1}$

5. $y = \sec(2x^3 + 3x - 1)^2$

6. $y = (\sin x)^{5x-2}$

Solución:

1. $y = (2x^2 + 3x - 1)^{16} \Longrightarrow y' = 16(2x^2 + 3x - 1)^{15}(4x + 3)$

2. $y = y = \ln\left(\dfrac{5x^2 + 2}{x^3 - 1}\right) \Longrightarrow y' = \dfrac{10x}{5x^2 + 2} - \dfrac{3x^2}{x^3 - 1}$

3. $y = x^3 \sec x \Longrightarrow y' = 3x^2 \sec x + x^3 \sec x \tan x$

4. $y = \dfrac{\cos x}{5x^2 - 1} \Longrightarrow y' = \dfrac{-\sin x \cdot (5x^2 - 1) - (10x)\cos x}{(5x^2 - 1)^2}$

5. $y = \sec(2x^3 + 3x - 1)^2 \Longrightarrow y' = 2(6x^2 + 3)(2x^3 + 3x - 1)\tan(2x^3 + 3x - 1)^2 \sec(2x^3 + 3x - 1)^2$

6. $y = (\sin x)^{5x-2} \Longrightarrow y' = (\sin x)^{5x-2}\left(5\ln(\sin x) + (5x - 2)\frac{\cos x}{\sin x}\right)$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{3x^2 + 4}{x^2 - 2}$ en el punto $x = 1$.

2. $f(x) = \dfrac{x^2 + 7}{2x - 1}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -7$ e $y - b = m(x - a)$

$$f'(x) = -\frac{20x}{(x^2 - 2)^2} \Longrightarrow m = f'(1) = -20$$

   Recta Tangente: $y + 7 = -20(x - 1)$

   Recta Normal: $y + 7 = \frac{1}{20}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -7$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(x^2 - x - 7)}{(2x - 1)^2} \Longrightarrow m = f'(0) = -14$$

   Recta Tangente: $y + 7 = -14x$

   Recta Normal: $y + 7 = \dfrac{1}{14}x$