Examen de Matemáticas 1º de Bachillerato

Noviembre 2006

Problema 1 Discutir y resolver por el método de Gauss los siguientes sistemas:

$$\left\{\begin{array}{rrrl} x- & y+ & z & = 1 \\ 2x+ & & z & = 1 \\ x+ & 3y- & z & = 5 \end{array}\right. \; ; \quad \left\{\begin{array}{rrrl} x- & y- & z & = 2 \\ 2x+ & y- & z & = 1 \\ x- & y+ & 2z & = 3 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrrl} x- & y+ & z & = 1 \\ 2x+ & & z & = 1 \\ x+ & 3y- & z & = 5 \end{array}\right. \text{ Sistema Incompatible} \Longrightarrow \text{No Tiene Solución}$$

$$\left\{\begin{array}{rrrl} x- & y- & z & = 2 \\ 2x+ & y- & z & = 1 \\ x- & y+ & 2z & = 3 \end{array}\right. \text{ Sistema Compatible Determinado} \Longrightarrow \left\{\begin{array}{l} x = 11/9 \\ y = -10/9 \\ z = 1/3 \end{array}\right.$$

Problema 2 Resolver las ecuaciones:

a) $\log(20x^2 + 10) - 1 = 2\log(x + 3)$

b) $3^{2x+1} + 3^{x-1} - 2 = 0$

c) $1 - \dfrac{1}{x^2 - 6x - 7} = \dfrac{x - 1}{x + 1} - \dfrac{1}{x - 7}$

d) $\dfrac{x^2 - 6x - 7}{x^2 - x - 6} \le 0$

e) $\sqrt{x + 3} + \sqrt{x + 2} = 2$

f) $\sqrt{2x - 1} - \sqrt{x - 1} = 1$

Solución:

a) $\log(20x^2 + 10) - 1 = 2\log(x + 3) \Longrightarrow \log \dfrac{20x^2 + 10}{10} = \log(x + 3)^2 \Longrightarrow$ $x^2 - 6x - 8 = 0 \Longrightarrow x = 7,1231, \;\; x = -1,1231.$

b) $3^{2x+1} + 3^{x-1} - 2 = 0 \Longrightarrow 3 \cdot t^2 + \dfrac{t}{3} - 2 = 0 \Longrightarrow 9t^2 + t - 6 = 0 \Longrightarrow t =$ $0,7628, \;\; t = -0,873$ (No Vale).

$3^x = 0,728 \Longrightarrow x = \dfrac{\log 0,7628}{\log 3} = -0,2465$

c) $1 - \dfrac{1}{x^2 - 6x - 7} = \dfrac{x - 1}{x + 1} - \dfrac{1}{x - 7} \Longrightarrow x = \dfrac{14}{3} = 4,67.$

d) $\dfrac{x^2-6x-7}{x^2-x-6} = \dfrac{(x+1)(x-7)}{(x+2)(x-3)} \leq 0 \Longrightarrow \ (-2,-1] \cup (3,7]$

e) $\sqrt{x+3}+\sqrt{x+2}=2 \Longrightarrow \ x=-\dfrac{23}{16}=x=-1,4375$

f) $\sqrt{2x-1}-\sqrt{x-1}=1 \Longrightarrow \ x=5, \quad x=1$